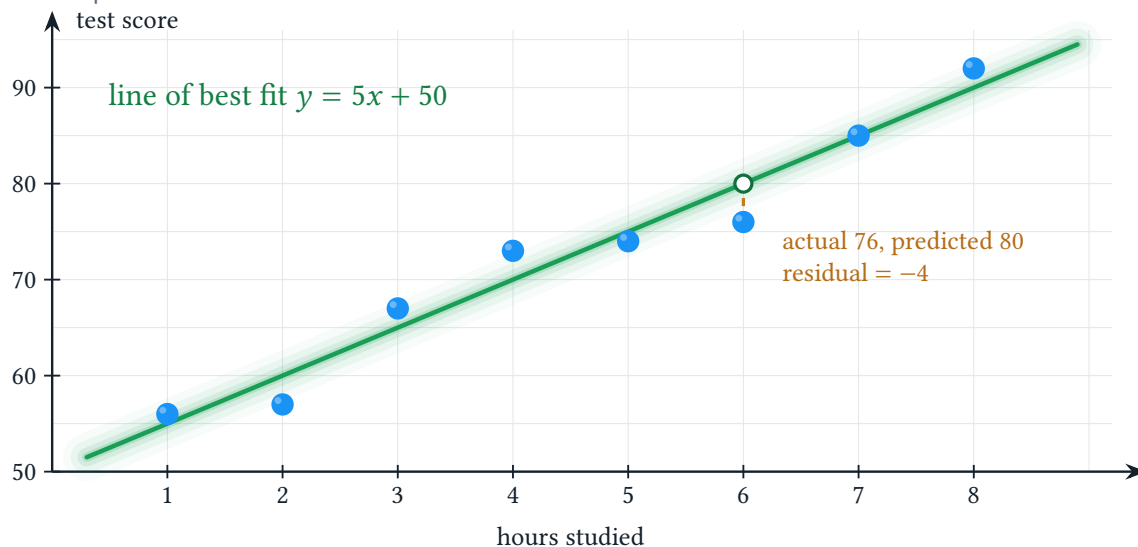


Problem-Solving & Data Analysis

Unit 3 Study Notes

Key ideas · Worked examples · Desmos tips

15 practice questions with full solutions



What these notes cover

Problem-Solving and Data Analysis is about 15% of SAT Math, roughly 5–7 of the 44 questions. It is real-world math: rates, percents and data. The College Board lists seven skills:

- 1 Ratios, rates, proportions and units
- 2 Percentages
- 3 One-variable data: center and spread
- 4 Two-variable data: models and scatterplots
- 5 Probability and conditional probability
- 6 Inference from samples and margin of error
- 7 Evaluating statistical claims

How to use them

Read a section, cover the solution to each worked example and try it yourself first. Then do the 15 practice questions in one sitting, with Desmos open, and check your work against the full solutions at the end.

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How the SAT tests this unit

These questions are wordy, and the math is usually light. The challenge is reading carefully: which quantity is the whole, which group the probability is taken from, and what a study can and cannot prove.

Skill	What the questions actually ask
Ratios, rates and units	Set up a proportion; convert units in a chain; compare unit rates.
Percentages	Percent of, percent change, successive changes, and finding the original before a change.
One-variable data	Mean, median, range and standard deviation; the effect of an outlier; read dot plots, histograms and box plots.
Two-variable data	Read a scatterplot; interpret the slope and intercept of a line of best fit; predict; tell linear from exponential.
Probability	Read a two-way table; find a probability, including “given that” (conditional) probability.
Inference and claims	Use a margin of error; decide whether a study supports a cause, and which population a result applies to.

THE BIG IDEA OF THIS UNIT

Always ask “**out of what?**” A rate is out of one unit of time or quantity, a percent is out of 100 parts of a whole, and a probability is out of the group you are choosing from. Most mistakes in this unit come from using the wrong whole.

1 Ratios, rates, proportions and units

A **rate** compares two quantities with different units, such as miles per gallon. A **proportion** says two ratios are equal, so you can scale one up or down.

UNITS ARE YOUR GUIDE

Write the units on every number and multiply by conversion factors that equal 1, such as $\frac{1000 \text{ m}}{1 \text{ km}}$, so that the unwanted units cancel. If the units of your answer are right, the setup is almost certainly right.

WORKED EXAMPLE 1.1

A car travels 150 miles on 6 gallons of gas. At this rate, how many gallons does it need for 425 miles?

Solution. Unit rate: $150 \div 6 = 25$ miles per gallon. Then $425 \div 25 = 17$ gallons.

WORKED EXAMPLE 1.2

Convert 72 kilometers per hour to meters per second.

Solution.

$$72 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = \frac{72\,000 \text{ m}}{3600 \text{ s}} = 20 \text{ m/s.}$$

WORKED EXAMPLE 1.3

On a map, 1 inch represents 15 miles. Two towns are 3.4 inches apart on the map. How far apart are they?

Solution. $3.4 \times 15 = 51$ miles.

2 Percentages

FOUR PERCENT TOOLS

Percent of	$p\% \text{ of } N \text{ is } \frac{p}{100} \times N.$
Percent change	$\frac{\text{new} - \text{original}}{\text{original}} \times 100\%.$ Divide by the <i>original</i> .
Multiplier	An increase of $r\%$ multiplies by $1 + \frac{r}{100}$; a decrease multiplies by $1 - \frac{r}{100}$.
Original value	If the new value is N after a change with multiplier k , the original was $N \div k.$

WORKED EXAMPLE 2.1

A price rises from \$80 to \$92. What is the percent increase?

Solution. $\frac{92 - 80}{80} = \frac{12}{80} = 0.15$, a 15% increase.

WORKED EXAMPLE 2.2

A price is increased by 20% and the new price is then decreased by 20%. What is the overall change?

Solution. Multiply the multipliers: $1.20 \times 0.80 = 0.96$. The final price is 96% of the original: a 4% decrease, not “no change”.

WORKED EXAMPLE 2.3

After a 25% discount, a jacket costs \$63. What was the original price?

Solution. A 25% discount leaves 75%, so $0.75 \times \text{original} = 63$ and the original price is $63 \div 0.75 = \$84$.

SAT TRAP

Successive percent changes do not add. Two changes of +20% and −20% do not cancel, because the second percent is taken of a different (larger) amount. Always multiply the multipliers.

3 One-variable data: center and spread**CENTER AND SPREAD**

Mean	The sum of the values divided by how many there are. Pulled toward outliers.
Median	The middle value when the values are in order (the mean of the two middle values if the count is even). Hardly moved by outliers.
Range	Largest value minus smallest value.
Standard deviation	How spread out the values are around the mean. You never calculate it on the SAT; you compare it: more spread out means larger.

WORKED EXAMPLE 3.1

Find the mean, median and range of 4, 7, 7, 9, 13. Then add the value 40 and find the new mean and median.

Solution. Mean = $\frac{40}{5} = 8$, median = 7, range = $13 - 4 = 9$.

With 40 added there are six values: the mean is $\frac{80}{6} \approx 13.3$, but the median is only $\frac{7+9}{2} = 8$. The outlier moved the mean by more than 5 and the median by just 1.

Reading a box plot

A box plot shows five numbers: the minimum, the first quartile Q_1 , the median, the third quartile Q_3 and the maximum. The box holds the middle half of the data. Its width, $Q_3 - Q_1$, is the **interquartile range**.

Scores: 62, 68, 71, 74, 75, 78, 80, 83, 85, 91, 96.

Median = 78; the lower half 62 ... 75 has median $Q_1 = 71$; the upper half 80 ... 96 has median $Q_3 = 85$.

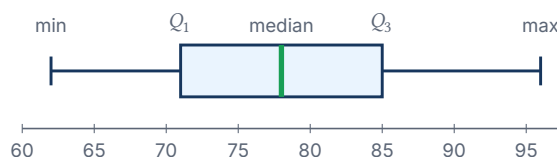


Figure 1. Box plot of the eleven scores.

SAT TRAP

A box plot shows *where* the data are, not how many values there are. Two box plots can look identical for data sets of very different sizes, and you cannot find the mean from a box plot.

4 Two-variable data: scatterplots and models

A scatterplot shows pairs of values. A **line of best fit** models the trend: its slope is the predicted change in y for each increase of 1 in x , and its y -intercept is the predicted value when $x = 0$.

WORKED EXAMPLE 4.1

The scatterplot on the cover shows hours studied, x , and test scores, y , for eight students, with line of best fit $y = 5x + 50$. Interpret the slope, and compare the student who studied 6 hours with the model.

Solution. The slope 5 means each extra hour of study is associated with a predicted increase of 5 points. For $x = 6$ the model predicts $5(6) + 50 = 80$, but the student scored 76. The **residual** (actual minus predicted) is $76 - 80 = -4$: the student scored 4 points *below* the prediction.

LINEAR OR EXPONENTIAL?

In a table with equal steps in x : if y goes up by the **same amount** each time, the model is linear; if y is multiplied by the **same factor** each time, it is exponential. For example, 3, 6, 12, 24 doubles each step, so it is exponential.

SAT TRAP

A line of best fit describes a trend in the data, not a promise for every individual, and it says nothing about cause. Also be careful predicting far outside the data: the model above would predict 100 points for 10 hours and 125 for 15, which is impossible.

5 Probability and two-way tables

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

A **conditional probability** (“given that...”) limits the outcomes to one row or one column of a table. That row or column total becomes the denominator.

Pet owned	Dog	Cat	None	Total
Grade 11	30	20	50	100
Grade 12	40	15	45	100
Total	70	35	95	200

Table 1. A survey of 200 students.

WORKED EXAMPLE 5.1

Using Table 1, find (a) the probability that a randomly chosen student owns a dog, (b) the probability that a student is in grade 12, given that the student owns a cat, and (c) the probability that a student owns a cat, given that the student is in grade 12.

Solution. (a) $\frac{70}{200} = 0.35$. (b) Only the 35 cat owners count, and 15 of them are in grade 12: $\frac{15}{35} = \frac{3}{7}$.
(c) Only the 100 grade-12 students count: $\frac{15}{100} = 0.15$.

Parts (b) and (c) use the same 15 students but different denominators, so the answers differ.

6 Samples, margin of error and study design

WHAT A SAMPLE CAN TELL YOU

Margin of error: a survey result of $p\%$ with margin of error $m\%$ means the true population value is *plausibly* between $(p - m)\%$ and $(p + m)\%$. Larger random samples give smaller margins of error.

Random sampling from a population lets you generalize to *that* population, and no further.

Random assignment to treatment groups (an experiment) lets you conclude *cause and effect*. An observational study, with no random assignment, shows only an association.

WORKED EXAMPLE 6.1

A random sample of 400 residents of a city found that 62% support a new park, with a margin of error of 4%. What can be concluded?

Solution. The percent of *all* residents of the city who support the park is plausibly between 58% and 66%. It is not certain to be exactly 62%, and the result says nothing about residents of other cities.

WORKED EXAMPLE 6.2

Researchers randomly assigned 100 volunteers to sleep either 6 or 8 hours a night for two weeks. The 8-hour group scored higher on a memory test. What can be concluded?

Solution. Because the groups were randomly assigned, the extra sleep *caused* the better scores, for people like these volunteers. Because the volunteers were not a random sample of all adults, the result should not be generalized to all adults.

7 Your Desmos toolkit for data

Desmos is not only a graphing calculator: it computes statistics and fits models to data. On the SAT that turns a page of arithmetic into one line.

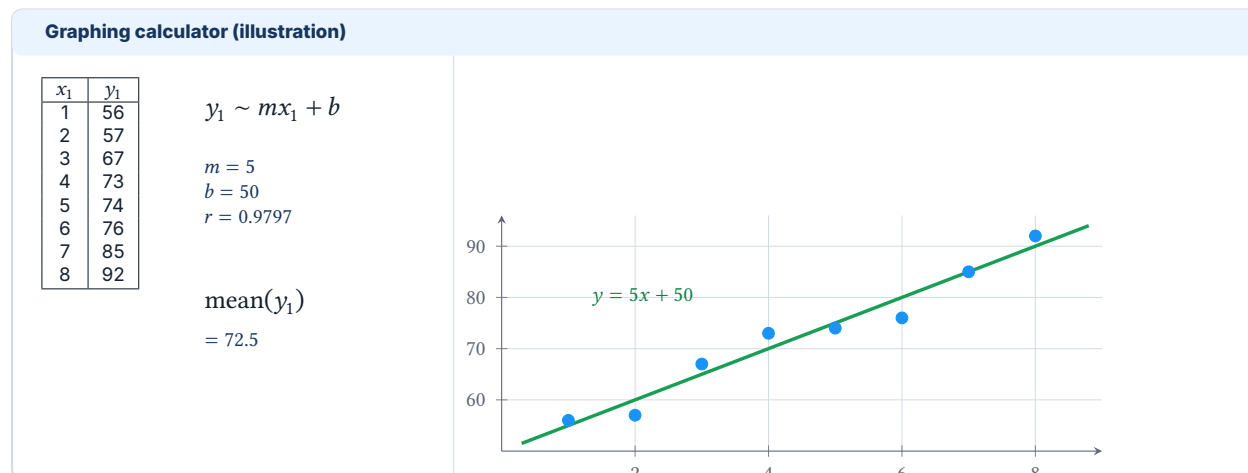


Figure 2. Type the data into a table, then $y_1 \sim mx_1 + b$: Desmos finds the line of best fit.

Five Desmos moves for data

1 · MEAN AND MEDIAN IN ONE LINE

Type a list: $L=[4,7,7,9,13]$, then $\text{mean}(L)$, $\text{median}(L)$ or $\text{stdev}(L)$. Edit the list to see what an outlier does.

2 · LINE OF BEST FIT

Add a table, type the data, then $y_1 \sim mx_1 + b$. Desmos reports m and b . Use $y_1 \sim ab^{x_1}$ for an exponential model.

3 · UNIT CONVERSIONS AS ONE EXPRESSION

Type the whole chain, for example $72 \cdot 1000 / 3600$, instead of working step by step. Fewer steps, fewer slips.

4 · PERCENT CHANGES AS MULTIPLIERS

Type $80 \cdot 1.15$ or $1.2 \cdot 0.8$ to check a percent change or a chain of changes instantly.

5 · PREDICTIONS FROM A MODEL

After a fit, type $5(6)+50$, or click the line at $x = 6$, to read the predicted value and compare it with the data.

SAT TRAP

Desmos gives you the numbers, not the meaning. It cannot tell you which total a probability is “out of”, or whether a study shows cause. Those are reading questions.

8 Practice questions

Fifteen original questions in the SAT style, arranged from easy to hard. Aim to finish in about 20 minutes, with Desmos open. Questions marked **HARD · MODULE 2** are the kind that separate good scores from top scores.

1 EASY

A printer prints 45 pages in 3 minutes. At this rate, how many pages does it print in 10 minutes?

Student-produced response: enter your answer.

2 EASY

What is 35% of 240?

- A** 68 **B** 84
C 96 **D** 205

3 EASY

What is the median of the data set 3, 5, 8, 8, 11?

- A** 5 **B** 7
C 8 **D** 11

4 EASY

A board is 4.5 feet long. How long is it in inches? (1 foot = 12 inches)

Student-produced response: enter your answer.

5 EASY

In a class of 30 students, 18 play a sport. If one student is chosen at random, what is the probability that the student plays a sport?

- A** $\frac{3}{10}$ **B** $\frac{2}{5}$
C $\frac{5}{3}$ **D** $\frac{3}{5}$

6 EASY

A shirt that costs \$40 is on sale for 15% off. What is the sale price, in dollars?

- A** 6 **B** 25
C 34 **D** 46

7 MEDIUM

The price of a bicycle increased from \$250 to \$290. By what percent did the price increase?

Student-produced response: enter your answer.

8 MEDIUM

The mean of five numbers is 12. Four of the numbers are 10, 14, 9 and 15. What is the fifth number?

Student-produced response: enter your answer.

9 MEDIUM

The line of best fit $y = 2.4x + 15$ models the cost y , in dollars, of an online order of x items. Which statement best interprets 2.4?

- A** The cost of 2.4 items is \$15.
B Each additional item increases the predicted cost by \$2.40.
C An order of no items costs \$2.40.
D Each additional item increases the predicted cost by \$15.

10 MEDIUM

A runner's speed is 15 meters per second. What is this speed in kilometers per hour?

- A** 54 **B** 4.2
C 25 **D** 900

11 MEDIUM

Of 80 customers at a cafe, 48 ordered coffee. Of the coffee customers, 30 also ordered a pastry, and 12 customers who did not order coffee ordered a pastry. If a customer who ordered a pastry is chosen at random, what is the probability that the customer also ordered coffee?

Student-produced response: enter your answer.

12 MEDIUM

The value 50 is added to the data set 12, 15, 15, 18, 20. Which statement is true?

- A** The mean increases more than the median.
- B** The median increases more than the mean.
- C** The mean and median increase by the same amount.
- D** Neither the mean nor the median changes.

13 HARD · MODULE 2

The price of a stock rises by 25% and then falls by 20%. How does the final price compare with the original price?

- A** 5% lower
- B** The same
- C** 5% higher
- D** 45% lower

14 HARD · MODULE 2

After a 30% decrease, the population of a town is 2,450. What was the population before the decrease?

Student-produced response: enter your answer.

15 HARD · MODULE 2

A random sample of 500 voters in a state found that 46% favor a proposal, with a margin of error of 4.4%. Which conclusion is most appropriate?

- A** Exactly 46% of the state's voters favor the proposal.
- B** Between 41.6% and 50.4% of the state's voters plausibly favor it.
- C** Most of the state's voters favor the proposal.
- D** Between 41.6% and 50.4% of all voters in the country favor it.

LEARN FROM EVERY MISTAKE

Before reading the solutions, mark each question you missed with the reason. Over time the pattern tells you exactly what to practice.

Reason	What it means	What to do
Concept	I did not know the rule or idea.	Re-read the section.
Method	I knew the idea but chose a slow or wrong approach.	Study the worked examples.
Misread	I used the wrong whole (for example, all customers instead of pastry customers).	Ask "out of what?"
Careless	An arithmetic or unit slip.	Check with Desmos.

9 Full solutions

1 150

Unit rate: $45 \div 3 = 15$ pages per minute, so $15 \times 10 = 150$ pages.

2 B

$$0.35 \times 240 = 84.$$

3 C

The values are already in order; the middle (third) value is 8.

4 54

$$4.5 \times 12 = 54 \text{ inches.}$$

5 D

$\frac{18}{30} = \frac{3}{5}$. Choice C is upside down: a probability is never greater than 1.

6 C

15% off leaves 85%: $0.85 \times 40 = 34$. Choice A, \$6, is the discount, not the price.

7 16

$\frac{290 - 250}{250} = \frac{40}{250} = 0.16$, a 16% increase. Divide by the original, 250, not by 290.

8 12

The five numbers total $5 \times 12 = 60$. The four known numbers total 48, so the fifth is $60 - 48 = 12$.

9 B

The slope is the change in predicted cost for each additional item. The 15 is the predicted fixed cost (the y -intercept).

10 A

$$15 \frac{\text{m}}{\text{s}} \times \frac{3600 \text{ s}}{1 \text{ h}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 54 \text{ km/h.}$$

11 5/7

The condition is “ordered a pastry”: $30 + 12 = 42$ customers. Of these, 30 ordered coffee: $\frac{30}{42} = \frac{5}{7} \approx 0.714$ (enter 5/7, .7142 or .7143).

12 A

Before: mean = $\frac{80}{5} = 16$, median = 15. After: mean = $\frac{130}{6} \approx 21.7$, median = $\frac{15+18}{2} = 16.5$. The mean rose by about 5.7, the median by 1.5.

13 B

$1.25 \times 0.80 = 1.00$, so the final price equals the original. The 20% fall is taken of a larger amount than the 25% rise.

14 3500

A 30% decrease leaves 70%: $0.70 \times P = 2450$, so $P = 2450 \div 0.70 = 3500$. Adding 30% back to 2,450 (giving 3,185) is the classic wrong answer.

15 B

The margin of error gives a plausible range for the whole population sampled: 46 ± 4.4 , from 41.6% to 50.4%, of voters *in that state*. A is too exact, C is not supported (the range is mostly below 50%), and D generalizes beyond the state.

Want to go further?

These notes are a sample of how I teach. In my live online SAT Math classes we work through all four units (Algebra, Advanced Math, Problem-Solving & Data Analysis, and Geometry & Trigonometry) in small groups, with full-length practice tests reviewed in class.

Book a free demo class: sat@joelsamuelacademy.in

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