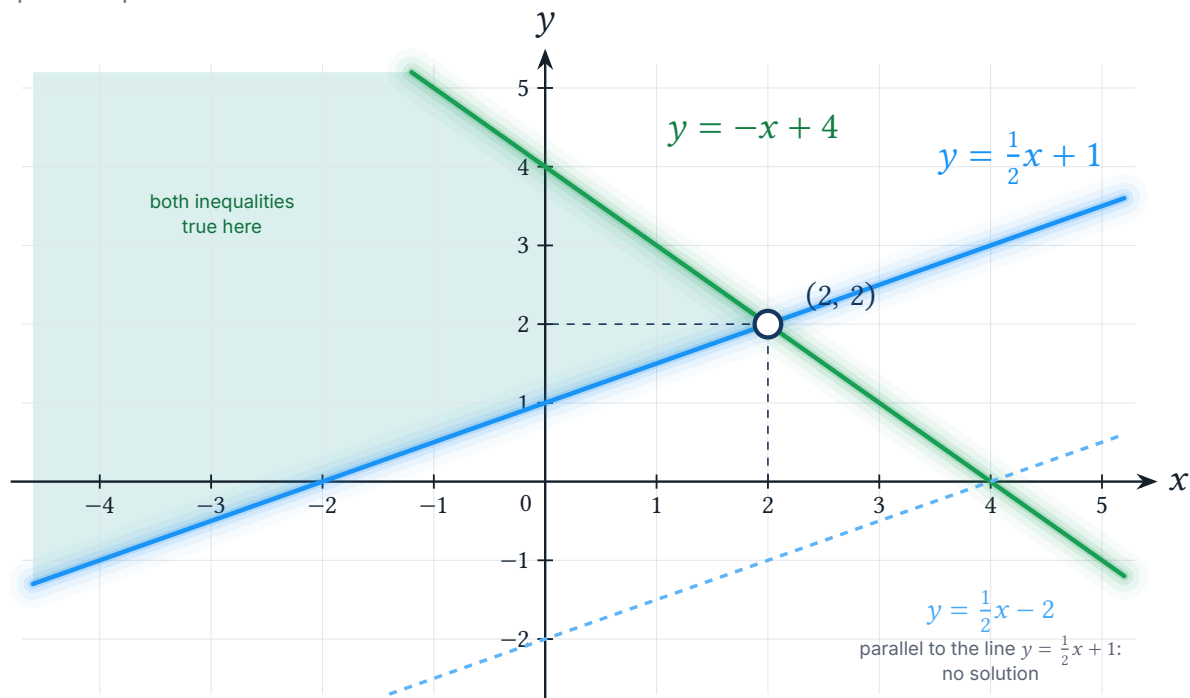


Algebra

Unit 1 Study Notes

Key ideas · Worked examples · Desmos tips
15 practice questions with full solutions



What these notes cover

Algebra is about 35% of SAT Math, roughly 13–15 of the 44 questions. Every question in this unit is about *linear* relationships, in five types:

- 1 Linear equations in one variable
- 2 Linear equations in two variables
- 3 Linear functions
- 4 Systems of two linear equations
- 5 Linear inequalities in one or two variables

How to use them

Read a section, cover the solution to each worked example and try it yourself first. Then do the 15 practice questions in one sitting, with Desmos open, and check your work against the full solutions at the end.

Contents

1	Linear equations in one variable	3
2	Linear equations in two variables and linear functions	4
3	Systems of two linear equations	5
4	Linear inequalities	6
5	Your Desmos toolkit for Algebra	8
6	Practice questions: 6 easy, 6 medium, 3 hard	9
7	Full solutions	11

How the SAT tests Algebra

The SAT rarely asks you simply to “solve for x ”. It asks you to *read* a linear situation: what a number in a model means, how many solutions an equation has, or which equation matches a graph, a table or a sentence. The table shows what each skill really tests.

Skill	What the questions actually ask
Equations in one variable	Solve efficiently; find an expression such as $3x + 1$ without finding x ; decide when an equation has no solution or infinitely many.
Equations in two variables	Turn a sentence into an equation; find a missing coordinate; interpret each number in context.
Linear functions	Slope as a rate of change, y-intercept as a starting value; move between equation, table and graph.
Systems of equations	Solve by substitution, elimination or Desmos; choose constants that give one, none or infinitely many solutions.
Linear inequalities	Translate “at least” and “at most”; flip the sign correctly; identify shaded regions and points that satisfy a system.

THE BIG IDEA OF THIS UNIT

A linear relationship changes by the **same amount** every time the input goes up by 1. That constant change is the **slope**. Almost every Algebra question on the SAT is a variation on this single idea.

1 Linear equations in one variable

An equation is a balance. Whatever you do to one side, you must do to the other, and the aim is to isolate the variable in as few clean steps as possible.

A RELIABLE ORDER OF STEPS

1. **Clear fractions** by multiplying every term by the least common denominator.
2. **Expand brackets** and combine like terms on each side.
3. **Collect** the variable terms on one side and the constants on the other.
4. **Divide** by the coefficient of the variable, then check by substituting back.

WORKED EXAMPLE 1.1

Solve $3(2x - 5) + 4 = 2x + 9$.

Solution. Expand: $6x - 15 + 4 = 2x + 9$, so $6x - 11 = 2x + 9$. Subtract $2x$ and add 11: $4x = 20$, so $x = 5$.

Check: $3(10 - 5) + 4 = 19$ and $2(5) + 9 = 19$. ✓

WORKED EXAMPLE 1.2

Solve $\frac{x+1}{3} - \frac{x-2}{4} = 2$.

Solution. The least common denominator is 12. Multiply every term by 12:

$$4(x+1) - 3(x-2) = 24 \implies 4x + 4 - 3x + 6 = 24 \implies x + 10 = 24 \implies x = 14.$$

Notice the bracket around $(x - 2)$: the minus sign in front multiplies *both* terms, giving $-3x + 6$.

How many solutions?

After simplifying, every linear equation in one variable looks like $ax + b = cx + d$. The coefficients decide everything:

Case	What happens	Number of solutions
$a \neq c$	The x -terms do not cancel, so you can solve for x .	exactly one
$a = c, b \neq d$	The x -terms cancel and you are left with a false statement such as $3 = 7$.	none
$a = c, b = d$	Both sides are identical, so every x works.	infinitely many

WORKED EXAMPLE 1.3

For what value of k does $4(x - 3) + kx = 7x - 12$ have infinitely many solutions?

Solution. Expand the left side: $(4 + k)x - 12 = 7x - 12$. The constants already match (-12 on both sides), so the equation is true for every x exactly when the coefficients match: $4 + k = 7$, so $k = 3$.

SAT TRAP

Read the last line of the question before you start. If you know $2x + 3 = 11$ and the question asks for $6x + 9$, do not stop at $x = 4$. Notice that $6x + 9 = 3(2x + 3) = 33$: often the fastest route skips finding x altogether.

2 Linear equations in two variables and linear functions

A linear function has the form $f(x) = mx + b$. Its graph is a straight line with **slope** m (the rate of change) and **y-intercept** b (the value when $x = 0$).

THREE FORMS OF A LINE, AND WHEN TO USE EACH

Slope-intercept	$y = mx + b$	Read the slope and y-intercept directly.
Point-slope	$y - y_1 = m(x - x_1)$	Write the line from one point and the slope.
Standard	$Ax + By = C$	Common in word problems; the slope is $-\frac{A}{B}$.
Slope between two points: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$. Parallel lines have equal slopes; perpendicular lines have slopes whose product is -1 .		

WORKED EXAMPLE 2.1

Find the equation of the line through $(-2, 5)$ and $(4, -7)$.

Solution. Slope: $m = \frac{-7 - 5}{4 - (-2)} = \frac{-12}{6} = -2$. Point-slope form with $(-2, 5)$: $y - 5 = -2(x + 2)$, so $y = -2x + 1$.

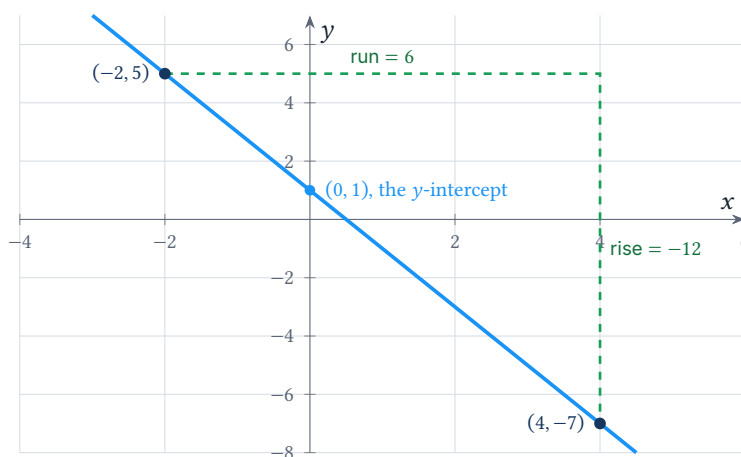


Figure 1. The line $y = -2x + 1$. Slope = rise $\div$ run = $-12 \div 6 = -2$.

Reading a linear model

Most Algebra questions describe a real situation. The skill being tested is interpretation: say what each number means, *with units*.

WORKED EXAMPLE 2.2

A tank is drained at a steady rate. The number of gallons left after t minutes is $V(t) = 480 - 12t$. Interpret 480 and 12, and find when the tank is empty.

Solution. 480 is the starting volume: there are 480 gallons when $t = 0$. The slope -12 means the volume *drops* by 12 gallons every minute. The tank is empty when $480 - 12t = 0$, that is, after $t = 40$ minutes.

WORKED EXAMPLE 2.3

A linear function f has $f(1) = 7$ and $f(4) = 19$. Find $f(10)$.

Solution. Rate of change: $\frac{19-7}{4-1} = 4$ per unit of x . From $x = 4$ to $x = 10$ is 6 more units, so $f(10) = 19 + 6 \cdot 4 = 43$. (In full, $f(x) = 4x + 3$.)

SAT TRAP

In a model such as $C = 35m + 50$, the number *multiplied by the variable* is the rate (here, \$35 per month) and the number standing alone is the fixed or starting amount (\$50). Answer choices often swap these two meanings.

3 Systems of two linear equations

The solution of a system is the point (x, y) that satisfies *both* equations: graphically, the point where the two lines meet. Two lines can meet once, never, or everywhere.

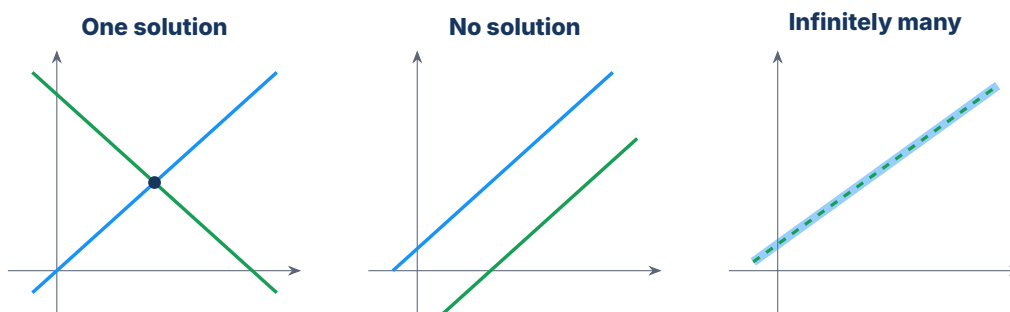


Figure 2. Different slopes: one solution. Same slope, different intercepts: none. Same line: infinitely many.

TWO METHODS, ONE TEST

Substitution works best when one variable is already alone ($y = \dots$). **Elimination** works best when the equations are in standard form: scale them so one variable has opposite coefficients, then add.

For $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$: if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ the lines are parallel or identical. They are **identical** (infinitely many solutions) if the constants are in the same ratio too, and **parallel** (no solution) if not.

WORKED EXAMPLE 3.1

Solve $3x + 2y = 16$ and $5x - 2y = 8$.

Solution. The y -terms are already opposite, so add the equations: $8x = 24$, giving $x = 3$. Then $3(3) + 2y = 16$, so $y = \frac{7}{2}$. The solution is $(3, \frac{7}{2})$.

WORKED EXAMPLE 3.2

A school sold 220 tickets to a play. Adult tickets cost \$12 and student tickets cost \$7, and ticket sales totaled \$2,040. How many student tickets were sold?

Solution. Let a and s be the numbers of adult and student tickets: $a + s = 220$ and $12a + 7s = 2040$. Substitute $a = 220 - s$:

$$12(220 - s) + 7s = 2040 \implies 2640 - 5s = 2040 \implies s = 120.$$

WORKED EXAMPLE 3.3 HARD · MODULE 2

In the system $2x + ky = 6$ and $3x - 6y = 9$, k is a constant. For what value of k does the system have infinitely many solutions?

Solution. For identical lines, the second equation must be a multiple of the first. Since $3 = 1.5 \times 2$ and $9 = 1.5 \times 6$, the multiplier is 1.5, so we need $1.5k = -6$, that is, $k = -4$.

If the first equation were $2x + ky = 5$ instead, the constants would *not* match ($1.5 \times 5 \neq 9$), and $k = -4$ would give **no** solution.

4 Linear inequalities

Inequalities are solved exactly like equations, with one extra rule.

THE ONE RULE TO REMEMBER

When you **multiply or divide both sides by a negative number**, reverse the inequality sign.

“at least”	$\geq$	“at most”, “no more than”	$\leq$
“more than”	$>$	“fewer than”, “less than”	$<$

WORKED EXAMPLE 4.1

Solve $-3x + 7 \leq 22$ and the compound inequality $-2 < 3x + 1 \leq 10$.

Solution. $-3x \leq 15$; dividing by -3 flips the sign: $x \geq -5$.

For the compound inequality, subtract 1 from all three parts, then divide by 3: $-3 < 3x \leq 9$, so $-1 < x \leq 3$.



Figure 3. $-1 < x \leq 3$: open circle (not included) at -1 , closed circle (included) at 3 .

Inequalities in two variables

The graph of $y > 2x - 1$ is the region *above* the line $y = 2x - 1$. The boundary is **dashed** for $<$ or $>$ (points on the line are not included) and **solid** for $\leq$ or $\geq$.

For a system of inequalities, the solutions are the points in the region where the shadings **overlap**. To test a point, substitute it into *every* inequality.

Example: is $(0, 2)$ a solution of $y > 2x - 1$ and $y \leq -x + 4$? We get $2 > -1$ ✓ and $2 \leq 4$ ✓, so yes.

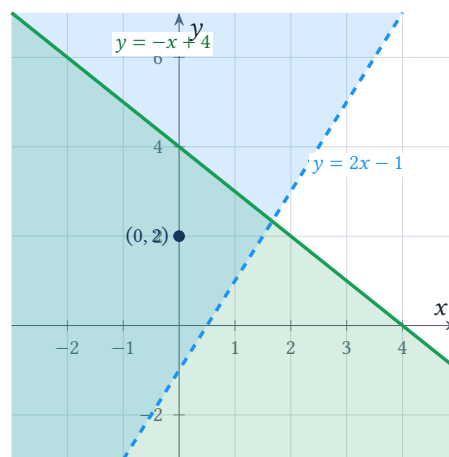


Figure 4. The solution set is the double-shaded wedge to the left of where the lines cross.

WORKED EXAMPLE 4.2

Notebooks cost \$3 each and pens cost \$1.50 each. Priya can spend at most \$30. Write an inequality for n notebooks and p pens. If she buys 6 notebooks, what is the greatest number of pens she can buy?

Solution. $3n + 1.5p \leq 30$. With $n = 6$: $18 + 1.5p \leq 30$, so $p \leq 8$. She can buy at most 8 pens.

SAT TRAP

In a word problem, the answer must make sense. If $p \leq 8.6$ for a number of pens, the greatest possible value is 8, not 9: round in the direction the inequality allows.

5 Your Desmos toolkit for Algebra

The Desmos graphing calculator is built into the digital SAT. For Algebra it is a fast way to *check* your work, and for some systems questions it is the fastest way to *solve*.

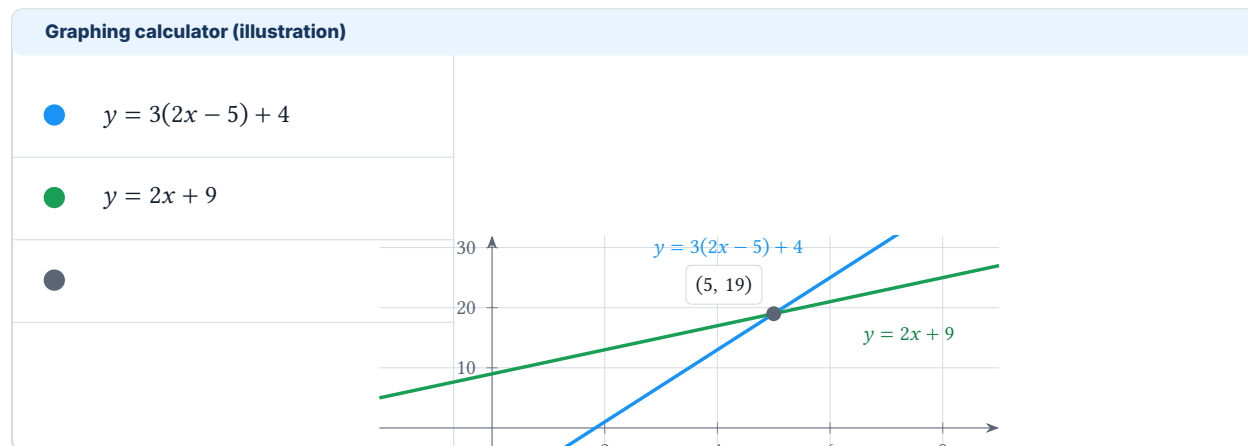


Figure 5. Solving $3(2x - 5) + 4 = 2x + 9$ by graphing each side: the lines meet at $x = 5$.

Five Desmos moves for Algebra

1 · SOLVE ANY ONE-VARIABLE EQUATION

Type the left side as $y = \dots$ and the right side as $y = \dots$, then click the intersection. Its x -coordinate is the solution (Figure 5).

2 · SYSTEMS: NO REARRANGING NEEDED

Type both equations exactly as given, for example $3x + 2y = 16$. Desmos graphs standard form directly. Click the intersection to read (x, y) .

3 · SPOT "NONE" OR "INFINITELY MANY"

Parallel lines that never meet: no solution. One line drawn on top of the other: infinitely many solutions.

4 · SLIDERS FOR UNKNOWN CONSTANTS

Type $2x + ky = 6$; Desmos offers a slider for k . Drag it until the lines become parallel or identical, then confirm with algebra.

5 · TEST ANSWER CHOICES QUICKLY

Type an inequality directly to see its shaded region, or type a point such as $(0, 2)$ to see whether it lies in the region.

SAT TRAP

Desmos shows decimals. If the answer choices are fractions, convert carefully (for example $3.5 = \frac{7}{2}$), and always re-read what the question asks for: x , y , or something like $x + y$.

6 Practice questions

Fifteen original questions in the SAT style, arranged from easy to hard. Aim to finish in about 20 minutes, with Desmos open. Questions marked **HARD · MODULE 2** are the kind that separate good scores from top scores.

1 EASY

If $5x - 8 = 3x + 10$, what is the value of x ?

- A** 1 **B** 4.5
C 9 **D** 18

2 EASY

If $2x + 3 = 11$, what is the value of $6x + 9$?

Student-produced response: enter your answer.

3 EASY

A line passes through $(0, -4)$ and $(2, 2)$. Which equation represents the line?

- A** $y = 3x - 4$ **B** $y = -4x + 3$
C $y = 3x + 4$ **D** $y = \frac{1}{3}x - 4$

4 EASY

The total cost, in dollars, of a gym membership for m months is $C(m) = 35m + 50$. What does 50 represent?

- A** The monthly fee
B The one-time sign-up fee
C The cost after 50 months
D The number of months

5 EASY

Which ordered pair (x, y) satisfies both $x + y = 9$ and $x - y = 3$?

- A** $(3, 6)$ **B** $(6, 3)$
C $(5, 4)$ **D** $(9, 0)$

6 EASY

Which of the following is the solution of $-4x + 6 > 18$?

- A** $x > -3$ **B** $x < -3$
C $x > 3$ **D** $x < -6$

7 MEDIUM

A linear function f has $f(2) = 11$ and $f(6) = 23$. What is the value of $f(0)$?

Student-produced response: enter your answer.

8 MEDIUM

Line ℓ is perpendicular to the line $y = \frac{2}{3}x + 1$ and passes through the point $(4, 1)$. What is the y -intercept of line ℓ ?

Student-produced response: enter your answer.

9 MEDIUM

If $4x - y = 10$ and $2x + 3y = 26$, what is the value of $x + y$?

Student-produced response: enter your answer.

10 MEDIUM

A caterer charges \$18 for each adult and \$11 for each child. An event with 60 guests cost \$947. How many of the guests were children?

- A** 19 **B** 23
C 37 **D** 41

11 MEDIUM

What value of x satisfies $\frac{x}{4} + \frac{x}{6} = 10$?

- A** 12 **B** 24
C 30 **D** 60

12 MEDIUM

Maya's first three test scores are 78, 85 and 91. What is the least score she needs on a fourth test so that the mean of her four scores is at least 85?

Student-produced response: enter your answer.

13 HARD · MODULE 2

In the equation $3(ax + 2) - 2x = 12x + 6$, a is a constant. If the equation has infinitely many solutions, what is the value of a ?

Student-produced response: enter your answer.

14 HARD · MODULE 2

In the system $kx + 3y = 5$ and $4x - 6y = 7$, k is a constant. For what value of k does the system have no solution?

- A** -4 **B** -2
C 2 **D** 4

15 HARD · MODULE 2

The graph of $y = mx + b$ is parallel to the line $3x - 2y = 8$ and passes through the point $(2, 9)$. At what value of x does the graph cross the x -axis?

Student-produced response: enter your answer.

LEARN FROM EVERY MISTAKE

Before reading the solutions, mark each question you missed with the reason. Over time the pattern tells you exactly what to practice.

Reason	What it means	What to do
Concept	I did not know the rule or idea.	Re-read the section.
Method	I knew the idea but chose a slow or wrong approach.	Study the worked examples.
Misread	I answered a different question (for example x instead of $x + y$).	Underline what is asked.
Careless	An arithmetic or sign slip.	Check with Desmos.

7 Full solutions**1 C**Subtract $3x$ and add 8: $2x = 18$, so $x = 9$.**2 33** $6x + 9 = 3(2x + 3) = 3(11) = 33$. No need to find x (which is 4).**3 A**Slope = $\frac{2-(-4)}{2-0} = 3$ and the y -intercept is -4 , so $y = 3x - 4$.**4 B**The 50 is not multiplied by m : it is paid once, whatever the number of months. The monthly fee is the rate, \$35.**5 B**Add the equations: $2x = 12$, so $x = 6$ and $y = 3$. Check: $6 + 3 = 9$ and $6 - 3 = 3$.**6 B** $-4x > 12$. Dividing by -4 reverses the sign: $x < -3$.**7 5**Slope = $\frac{23-11}{6-2} = 3$. Going from $x = 2$ back to $x = 0$ subtracts 2×3 : $f(0) = 11 - 6 = 5$.**8 7**Perpendicular slope: $-\frac{3}{2}$ (negative reciprocal of $\frac{2}{3}$). Then $1 = -\frac{3}{2}(4) + b$, so $b = 7$.**9 10**From the first equation, $y = 4x - 10$. Substitute: $2x + 3(4x - 10) = 26$, so $14x = 56$, $x = 4$, $y = 6$, and $x + y = 10$.*Desmos*: type both equations and click the intersection, $(4, 6)$.**10 A**Let c be the number of children: $18(60 - c) + 11c = 947$, so $1080 - 7c = 947$ and $c = 19$.Choice D, 41, is the number of *adults*: a classic trap.**11 B**Multiply by 12: $3x + 2x = 120$, so $5x = 120$ and $x = 24$.**12 86**The mean is at least 85 when the total is at least $4 \times 85 = 340$. The first three scores total 254, so she needs $x \geq 340 - 254 = 86$.**13 14/3**Expand: $(3a - 2)x + 6 = 12x + 6$. The constants match, so for infinitely many solutions $3a - 2 = 12$, giving $a = \frac{14}{3}$ (enter $14/3$, or the decimal 4.666 or 4.667).**14 B**No solution means parallel lines: equal slopes, different intercepts. Slope of $4x - 6y = 7$ is $\frac{4}{6} = \frac{2}{3}$; slope of $kx + 3y = 5$ is $-\frac{k}{3}$. So $-\frac{k}{3} = \frac{2}{3}$ and $k = -2$. Check the intercepts: $\frac{5}{3} \neq -\frac{7}{6}$, so the lines are distinct and parallel. ✓**15 -4**Rewrite $3x - 2y = 8$ as $y = \frac{3}{2}x - 4$: the slope is $\frac{3}{2}$. The parallel line through $(2, 9)$: $9 = \frac{3}{2}(2) + b$, so $b = 6$ and $y = \frac{3}{2}x + 6$. It crosses the x -axis where $y = 0$: $\frac{3}{2}x = -6$, so $x = -4$.**Want to go further?**

These notes are a sample of how I teach. In my live online SAT Math classes we work through all four units (Algebra, Advanced Math, Problem-Solving & Data Analysis, and Geometry & Trigonometry) in small groups, with full-length practice tests reviewed in class.

Book a free demo class: sat@joelsamuelacademy.in

joel samuel
ACADEMY
www.joelsamuelacademy.in