joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 9. $2x + \frac{7}{2} > \frac{5x}{3} + 3, x \in I.$

$2x + \frac{7}{2} > \frac{5x}{3} + 3 \quad ; \quad x \in I$

$2x - \frac{5x}{3} > 3 - \frac{7}{2}$

$\frac{6x - 5x}{3} > \frac{6 - 7}{2}$

$\frac{x}{3} > \frac{-1}{2}$

$x > \frac{-3}{2}$

$x > -1.5$

Solution Set = $\{-1, 0, 1, 2, \cdots\}$



joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 10. $2x + 3 \leq 3x + 1, x \in R$

$2x + 3 \leq 3x + 1, \quad x \in \mathbb{R}$

$3 - 1 \leq 3x - 2x$

$2 \leq x$

$x \geq 2$

Solution set $= \{x : x \geq 2, x \in \mathbb{R}\}$

⇓

To be read as "Set of all x such that x is ≥ 2 & x belongs to Real Nos"



Note: In case the Replacement set or Domain is Real Nos $\mathbb{R}$, then

(i) Solution set can only be given in set builder notation.

(ii) A thick continuous line represents the soln on the Real No. Line

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 11. $\frac{5x-8}{3} \geq \frac{4x-7}{2}, x \in R$

$$\frac{5x-8}{3} \geq \frac{4x-7}{2} \; ; \; x \in \mathbb{R}$$

$$2(5x-8) \geq 3(4x-7)$$

$$10x-16 \geq 12x-21$$

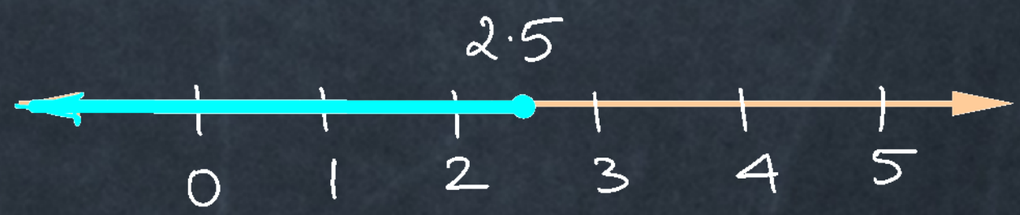
$$21-16 \geq 12x-10x$$

$$5 \geq 2x$$

$$2.5 \geq x$$

$$x \leq 2.5$$

Solution set $\{x : x \in \mathbb{R}, x \leq 2.5\}$



joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Solve the following equations factorization :

Q. 3. $16x^2 = 25$

$16x^2 = 25$

$16x^2 - 25 = 0$

$(4x)^2 - (5)^2 = 0$

$(4x-5)(4x+5) = 0$

$a^2 - b^2 = 0$

$(a-b)(a+b) = 0$

For $4x - 5 = 0 \Rightarrow x = 5/4$

For $4x + 5 = 0 \Rightarrow x = -5/4$

Roots $\left\{-\frac{5}{4}, \frac{5}{4}\right\}$

Q. 4. $x^2 + 2x = 24$

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

Sum $= +2x$

Prod. $= -24x^2$ < 24×1, 12×2, 6×4

$x(x+6) - 4(x+6) = 0$

$(x-4)(x+6) = 0$

For $x - 4 = 0 \Rightarrow x = 4$

$x + 6 = 0 \Rightarrow x = -6$

Roots $\{-6, 4\}$

$ax^2 + bx + c = 0$

X Y

Sum $X + Y = +bx$

Prod. $XY = +acx^2$

Zero Product Rule

If $ab = 0$ then either $a = 0$ or $b = 0$

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 5. $x^2 - x = 156$

$x^2 - x - 156 = 0$

Sum $= -x$

Prod $= -156x^2$

$x^2 - 13x + 12x - 156 = 0$

$13 \times 3 \times 2 \times 2$

12

$x(x-13) + 12(x-13) = 0$

$(x+12)(x-13) = 0$

For $x + 12 = 0 \Rightarrow x = -12$

$x - 13 = 0 \Rightarrow x = 13$

$\therefore$ Roots $\{-12, 13\}$

$156 = 2 \times 2 \times 3 \times 13$

Q. 6. $x^2 - 11x = 42$

$x^2 - 11x - 42 = 0$

$x^2 - 14x + 3x - 42 = 0$

$x(x-14) + 3(x-14) = 0$

$(x+3)(x-14) = 0$

For $x + 3 = 0 \Rightarrow x = -3$

$x - 14 = 0 \Rightarrow x = 14$

Roots $\{-3, 14\}$

Sum $= -11x$

Prod $= -42x^2$

$42 = 2 \times 3 \times 7$

6×7

2×21

14×3

Q. 7. $x^2 - 7x + 10 = 0$

$x^2 - 5x - 2x + 10 = 0$

$x(x-5) - 2(x-5) = 0$

$(x-2)(x-5) = 0$

For $x - 2 = 0 \Rightarrow x = 2$

$x - 5 = 0 \Rightarrow x = 5$

Roots $\{2, 5\}$

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

* Concept

When $\sqrt{ax+b} = cx+d$; for which if α, β are roots

Select only roots for which LHS = RHS.

Q. 42. $\sqrt{x+15} = (x+3)$

Squaring both sides

$$\left(\sqrt{x+15}\right)^2 = (x+3)^2$$

$$x+15 = x^2+9+6x$$

$$x^2+5x-6=0$$

$$x^2+6x-x-6=0$$

$$x(x+6)-1(x+6)=0$$

$$(x-1)(x+6)=0$$

For $x-1=0 \Rightarrow x=1$

$x+6=0 \Rightarrow x=-6$

$$\left(\sqrt{x}\right)^2 = x$$

For $x=1$

$LHS = \sqrt{1+15} = \sqrt{16} = 4$

$RHS = 1+3 = 4$

$LHS = RHS$

For $x=-6$

$LHS = \sqrt{-6+15} = \sqrt{9} = 3$

$RHS = -6+3 = -3$

$LHS \neq RHS$

$\therefore$ Root $\{1\}$

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Find the values of k for which each of the following equations has equal roots.

Q. 13. $9x^2 + kx + 1 = 0$ $\longrightarrow$ $a = 9$, $b = k$ $c = 1$

For equal roots, we know $b^2 - 4ac = 0$

$$(k)^2 - 4(9)(1) = 0$$

$$k^2 = 36 \Rightarrow k = \sqrt{36}$$

$$\boxed{k = \pm 6}$$

Q. 14. $x^2 - 2kx + 7k - 12 = 0$ $a = 1$, $b = -2k$, $c = (7k - 12)$

For equal roots $b^2 - 4ac = 0$

$$(-2k)^2 - 4(1)(7k - 12) = 0$$

$$4k^2 - 28k + 48 = 0$$

Divide by 4 throughout $\Rightarrow$

$$k^2 - 7k + 12 = 0$$

$$k^2 - 4k - 3k + 12 = 0$$

$$k(k-4) - 3(k-4) = 0$$

$$(k-3)(k-4) = 0$$

For $k - 3 = 0 \Rightarrow k = 3$

$k - 4 = 0 \Rightarrow k = 4$

$\therefore$ $\boxed{k = 3, 4}$

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 12. A two-digit number contains the smaller of the two digits in the unit place. The product of the digits is 40 and the difference between the digits is 3. Find the number.

Let the No. be $\underset{x}{\underline{I}}\ \underset{y}{\underline{u}}$; $x > y$

Product $(x)\cdot(y) = 40$ — (i)

$$y = \frac{40}{x} \quad \text{— (ii)}$$

Diff. of digits : $x - y = 3$ ——— (iii)

Substitute (ii) in (iii) $\quad x - \frac{40}{x} = 3$

$$x^2 - 3x - 40 = 0$$

$$x^2 - 8x + 5x - 40 = 0$$

$$x(x-8) + 5(x-8) = 0$$

$$(x+5)(x-8) = 0$$

For $x + 5 = 0 \Rightarrow x = -5$ (Neglect)

$x - 8 = 0 \Rightarrow x = 8$

From (ii) $\quad y = \frac{40}{8} = 5$

$\therefore$ The Number is 85

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 15. If $x=\dfrac{\sqrt{b+3a}+\sqrt{b-3a}}{\sqrt{b+3a}-\sqrt{b-3a}}$, prove that $3ax^2 - 2bx + 3a = 0$.

$$\frac{x}{1} = \frac{\sqrt{b+3a}+\sqrt{b-3a}}{\sqrt{b+3a}-\sqrt{b-3a}}$$

By Componendo Dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt{b+3a}+\sqrt{b-3a}+\sqrt{b+3a}-\sqrt{b-3a}}{\sqrt{b+3a}+\sqrt{b-3a}-\sqrt{b+3a}+\sqrt{b-3a}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{b+3a}}{2\sqrt{b-3a}} \Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{b+3a}}{\sqrt{b-3a}}$$

Square both sides : $\dfrac{(x+1)^2}{(x-1)^2} = \dfrac{b+3a}{b-3a} \Rightarrow \dfrac{x^2+2x+1}{x^2-2x+1} = \dfrac{b+3a}{b-3a}$

$$(b-3a)(x^2+2x+1) = (b+3a)(x^2-2x+1)$$

$$bx^2+2bx+b-3ax^2-6ax-3a = bx^2-2bx+b+3ax^2-6ax+3a$$

$$0 = 6ax^2-4bx+6a$$

Divide by 2 throughout

$$\boxed{3ax^2-2bx+3a=0}$$

Hence proved

joel samuel
9182099001

joel samuel
ACADEMY
www.joelsamuelacademy.in

Q. 16. If $x=\dfrac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$, prove that:

$$3bx^2 - 4ax + 3b = 0.$$

Q. 17. If $x = \dfrac{\sqrt[3]{m+1}+\sqrt[3]{m-1}}{\sqrt[3]{m+1}-\sqrt[3]{m-1}}$, prove that

$$x^3 - 3x^2m + 3x - m = 0.$$

HW.

$$\frac{x}{1} = \frac{\sqrt[3]{m+1}+\sqrt[3]{m-1}}{\sqrt[3]{m+1}-\sqrt[3]{m-1}}$$

By Componendo Dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt[3]{m+1}+\sqrt[3]{m-1}+\sqrt[3]{m+1}-\sqrt[3]{m-1}}{\sqrt[3]{m+1}+\sqrt[3]{m-1}-\sqrt[3]{m+1}+\sqrt[3]{m-1}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt[3]{m+1}}{2\sqrt[3]{m-1}} \Rightarrow \frac{x+1}{x-1} = \frac{\sqrt[3]{m+1}}{\sqrt[3]{m-1}}$$

Cubing both sides

$$\frac{(x+1)^3}{(x-1)^3} = \frac{m+1}{m-1} \Rightarrow \frac{x^3+3x^2+3x+1}{x^3-3x^2+3x-1} = \frac{m+1}{m-1}$$

$$(m-1)(x^3+3x^2+3x+1) = (m+1)(x^3-3x^2+3x-1)$$

$$mx^3+3mx^2+3mx+m-x^3-3x^2-3x-1 =$$

$$mx^3-3mx^2+3mx-m+x^3-3x^2+3x-1$$

$$0 = 2x^3-6mx^2+6x-2m \Rightarrow \boxed{x^3-3mx^2+3x-m=0}$$

Hence Proved

joel samuel
9182099001