

UNIT V

STATISTICS AND PROBABILITY

13. Statistics

14. Probability

STATISTICS

Mean Deviation for Ungrouped Data

$$MD(\bar{x}) = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$$

$\bar{x} \rightarrow$ Mean

$$MD(M) = \frac{1}{n} \sum_{i=1}^n |x_i - M|$$

$M \rightarrow$ Median

Example 1 Find the mean deviation about the mean for the following data:

6, 7, 10, 12, 13, 4, 8, 12

Data : 6, 7, 10, 12, 13, 4, 8, 12

$$\bar{x} = \frac{\sum x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$$

$$\bar{x} = 9$$

$$MD(\bar{x}) = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$$

$$|x_i - \bar{x}| = 3, 2, 1, 3, 4, 5, 1, 3$$

$$MD(\bar{x}) = \frac{1}{8} \sum (3+2+1+3+4+5+1+3) = \frac{22}{8}$$

$$MD(\bar{x}) = 2.75$$

Find the mean deviation about the median for the data in Exercises 3 and 4.

3. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

4. 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

$$MD(M) = \frac{\sum |x_i - M|}{n}$$

③ 10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$n = 12 \text{ (even)} \Rightarrow \text{Median} = \frac{1}{2} \left[\frac{n}{2}^{\text{th}} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \right] \text{ terms} = \frac{1}{2} \left[6^{\text{th}} \& 7^{\text{th}} \right] \text{ terms}$$

$$\text{Median} = \frac{1}{2} (13 + 14) \Rightarrow M = 13.5$$

$$\sum |x_i - M| = 3.5 + 2.5 + 2.5 + 1.5 + 0.5 + 0.5 + 0.5 + 2.5 + 2.5 + 3.5 + 3.5 + 4.5$$

$$\sum |x_i - M| = 28 \Rightarrow MD(M) = \frac{\sum |x_i - M|}{n} = \frac{28}{12} = 2.33$$

$$MD(M) = 2.33$$

95
94
47.5

④ 36, 42, 45, 46, 46, 49, 51, 53, 60, 72

$$n = 10 \text{ (even)} \quad \text{Median} = \frac{1}{2} \left[\frac{n}{2}^{\text{th}} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \right] \text{ terms} = \frac{1}{2} \left[5^{\text{th}} + 6^{\text{th}} \right] \text{ terms}$$

$$M = \frac{1}{2} (46 + 49) = 47.5 \Rightarrow M = 47.5$$

$$\sum |x_i - M| = 11.5 + 5.5 + 2.5 + 1.5 + 1.5 + 1.5 + 3.5 + 5.5 + 12.5 + 24.5 = 70$$

$$MD(M) = \frac{\sum |x_i - M|}{n} = \frac{70}{10} = 7$$

$$MD(M) = 7$$

Find the mean deviation about the mean for the data in Exercises 9 and 10.

9. Income per day in ₹	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

$$MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

$$\bar{x} = \frac{\sum f_i d_i}{\sum f_i}$$

9.

CI	Class mark x_i	f_i	$A = 350$ $d_i = x_i - A$	$f_i d_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
0-100	50	4	-300	-1200	308	1232
100-200	150	8	-200	-1600	208	1664
200-300	250	9	-100	-900	108	972
300-400	350	10	0	0	8	80
400-500	450	7	100	700	92	644
500-600	550	5	200	1000	192	960
600-700	650	4	300	1200	292	1168
700-800	750	3	400	1200	328	1172
	$\sum f_i = 50$			$\sum f_i d_i = 400$		$\sum f_i x_i - \bar{x} = 7896$

Mean

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 350 + \frac{400}{50} = 358$$

$$\bar{x} = 358$$

$$MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{7896}{50} = 157.92$$

$$MD(\bar{x}) = 157.92$$

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

7.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

$\bar{x}, \sigma^2$

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

CI	x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
0-30	15	2	30	-92	8464	16928
30-60	45	3	135	-62	3844	11532
60-90	75	5	375	-32	1024	5120
90-120	105	10	1050	-2	4	40
120-150	135	3	405	28	784	2352
150-180	165	5	825	58	3364	16820
180-210	195	2	390	88	7744	15488

$$N = \sum f_i = 30 \quad \sum f_i x_i = 3210$$

$$\sum f_i (x_i - \bar{x})^2 = 68280$$

Mean ($\bar{x}$)

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3210}{30} = 107$$

$$\bar{x} = 107$$

Variance

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \frac{68280}{30} = 2276 \quad \therefore$$

$$\sigma^2 = 2276$$

9. Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

CI	x_i	f_i	$A = 92.5$ $y_i = \frac{x_i - 92.5}{5}$	$f_i y_i$	$f_i y_i^2$
70-75	72.5	3	-4	-12	48
75-80	77.5	4	-3	-12	36
80-85	82.5	7	-2	-14	28
85-90	87.5	7	-1	-7	7
90-95	92.5	15	0	0	0
95-100	97.5	9	1	9	9
100-105	102.5	6	2	12	24
105-110	107.5	6	3	18	54
110-115	112.5	3	4	12	48

$$N = \sum f_i = 60$$

$$\sum f_i y_i = 6 \quad \sum f_i y_i^2 = 254$$

$$\sigma^2 = \frac{h^2}{N^2} \left[N \sum f_i y_i^2 - (\sum f_i y_i)^2 \right]$$

$$\bar{x} = A + h \frac{\sum f_i y_i}{\sum f_i} ; h = 5$$

Mean

$$\begin{aligned} \bar{x} &= A + h \frac{\sum f_i y_i}{\sum f_i} \\ &= 92.5 + 5 \left(\frac{6}{60} \right) \end{aligned}$$

$$\bar{x} = 93$$

Variance

$$\sigma^2 = \frac{h^2}{N^2} \left[N \sum f_i y_i^2 - (\sum f_i y_i)^2 \right]$$

$$\sigma^2 = \frac{25}{3600} \left[60(254) - 36 \right]$$

$$\sigma^2 = 105.58$$

Std. Deviation

$$\sigma = \sqrt{\sigma^2} = \sqrt{105.58} \Rightarrow \sigma = 10.27$$

PROBABILITY

Example 1 Two coins (a one rupee coin and a two rupee coin) are tossed once. Find a sample space.

$$S = \{HH, HT, TH, TT\}$$

Example 2 Find the sample space associated with the experiment of rolling a pair of dice (one is blue and the other red) once. Also, find the number of elements of this sample space.

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}$$

$$n(S) = 36$$

EXERCISE 16.2

1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{4\} \quad F = \{2, 4, 6\}$$

$$E \cap F = \{4\} \neq \emptyset \quad \text{No } E \& F \text{ are NOT mutually exclusive.}$$

2. A die is thrown. Describe the following events:

- | | |
|--------------------------------------|----------------------------------|
| (i) A: a number less than 7 | (ii) B: a number greater than 7 |
| (iii) C: a multiple of 3 | (iv) D: a number less than 4 |
| (v) E: an even number greater than 4 | (vi) F: a number not less than 3 |

Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F'$, F'

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$(i) \quad A = \{1, 2, 3, 4, 5, 6\}$$

$$(ii) \quad B = \emptyset$$

$$(iii) \quad C = \{3, 6\}$$

$$(iv) \quad D = \{1, 2, 3\}$$

$$(v) \quad E = \{6\}$$

$$(vi) \quad F = \{3, 4, 5, 6\}$$

$$F' = \{1, 2\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$B \cup C = \{3, 6\}$$

$$D \cap E = \emptyset$$

$$A \cap B = \emptyset$$

$$E \cap F = \{6\}$$

$$A - C = \{1, 2, 4, 5\}$$

$$D - E = \{1, 2, 3\}$$

$$E \cap F' = \emptyset$$

$$F' = \{1, 2\}$$

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8, B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}$$

$$\begin{array}{cccc} \textcircled{9} & 10 & 11 & \textcircled{12} \end{array}$$

$$A = \left\{ \overbrace{(3,6) (4,5) (5,4) (6,3)}^{\textcircled{9}} \quad \overbrace{(4,6) (5,5) (6,4)}^{\textcircled{10}} \quad \overbrace{(5,6) (6,5)}^{\textcircled{11}} \quad \overbrace{(6,6)}^{\textcircled{12}} \right\}$$

$$B = \left\{ (1,2) (2,2) (3,2) (4,2) (5,2) (6,2) \right. \\ \left. (2,1) (2,3) (2,4) (2,5) (2,6) \right\}$$

$$C = \left\{ (3,6) (4,5) (5,4) (6,3) (6,6) \right\}$$

$$A \cap B = \emptyset$$

$$B \cap C = \emptyset$$

$$A \cap C = \left\{ (3,6) (4,5) (5,4) (6,3) (6,6) \right\} \neq \emptyset$$

A & B are mutually exclusive
B & C are mutually exclusive

Example 11 A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the probability that it will be (i) red, (ii) yellow, (iii) blue, (iv) not blue, (v) either red or blue.

4R, 3B
2Y

E: Getting red

$$n(E) = 4$$

F: Getting blue

$$n(F) = 3$$

G: Getting yellow

$$n(G) = 2$$

$$n(S) = 9$$

$$(i) P(E) = \frac{4}{9}$$

$$(ii) P(G) = \frac{2}{9}$$

$$(iii) P(F) = \frac{3}{9} = \frac{1}{3}$$

$$(iv) P(F') = 1 - P(F) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$(v) P(E \cup F) = P(E) + P(F)$$

$$= \frac{4}{9} + \frac{1}{3} = \frac{4+3}{9} = \frac{7}{9}$$

Example 12 Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

- Both Anil and Ashima will not qualify the examination.
- Atleast one of them will not qualify the examination and
- Only one of them will qualify the examination.

Let

E : Anil qualifies

F : Ashima qualifies

Given: $P(E) = 0.05$, $P(F) = 0.1$, $P(E \cap F) = 0.02$

$$\begin{aligned} \text{a) } P(\text{Not } E \text{ \& Not } F) &= P(E' \cap F') \\ &= P(E \cup F)' \end{aligned}$$

De Morgan's Law

$$E' \cap F' = (E \cup F)'$$

$P($

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= 0.05 + 0.1 - 0.02$$

$$P(E \cup F) = 0.13$$

$$\begin{aligned} 1 - P(E \cup F) &= 1 - 0.13 \\ &= 0.87 \end{aligned}$$

$$\therefore P(E' \cap F') = 0.87$$