

UNIT III

COORDINATE GEOMETRY

9. Straight Lines

10. Conic Sections

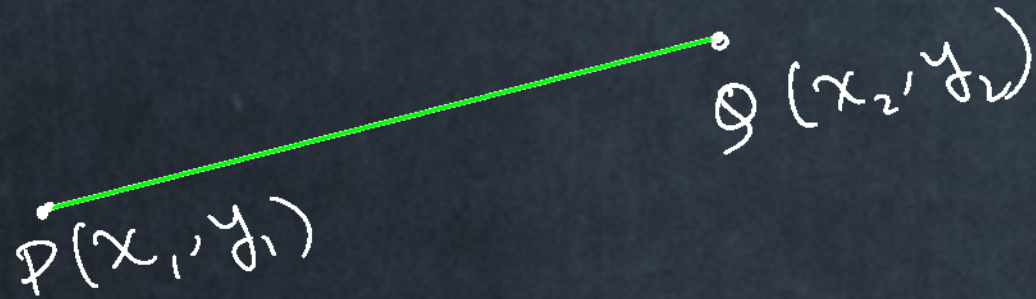
11. Introduction to 3D Geometry

STRAIGHT LINES

Revision of Coordinate Geometry

① Distance formula

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

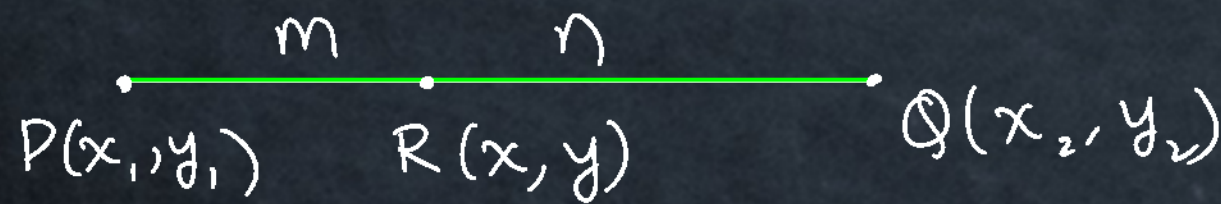


② Section formula

- a) Internal Division
- b) External Division*

Mid point formula

a) Internal Division



$$x = \frac{mx_2 + nx_1}{m+n}; \quad y = \frac{my_2 + ny_1}{m+n}$$

b) External Division



$$\frac{PR}{QR} = \frac{m}{n}$$

$$x = \frac{mx_2 - nx_1}{m-n}; \quad y = \frac{my_2 - ny_1}{m-n}$$

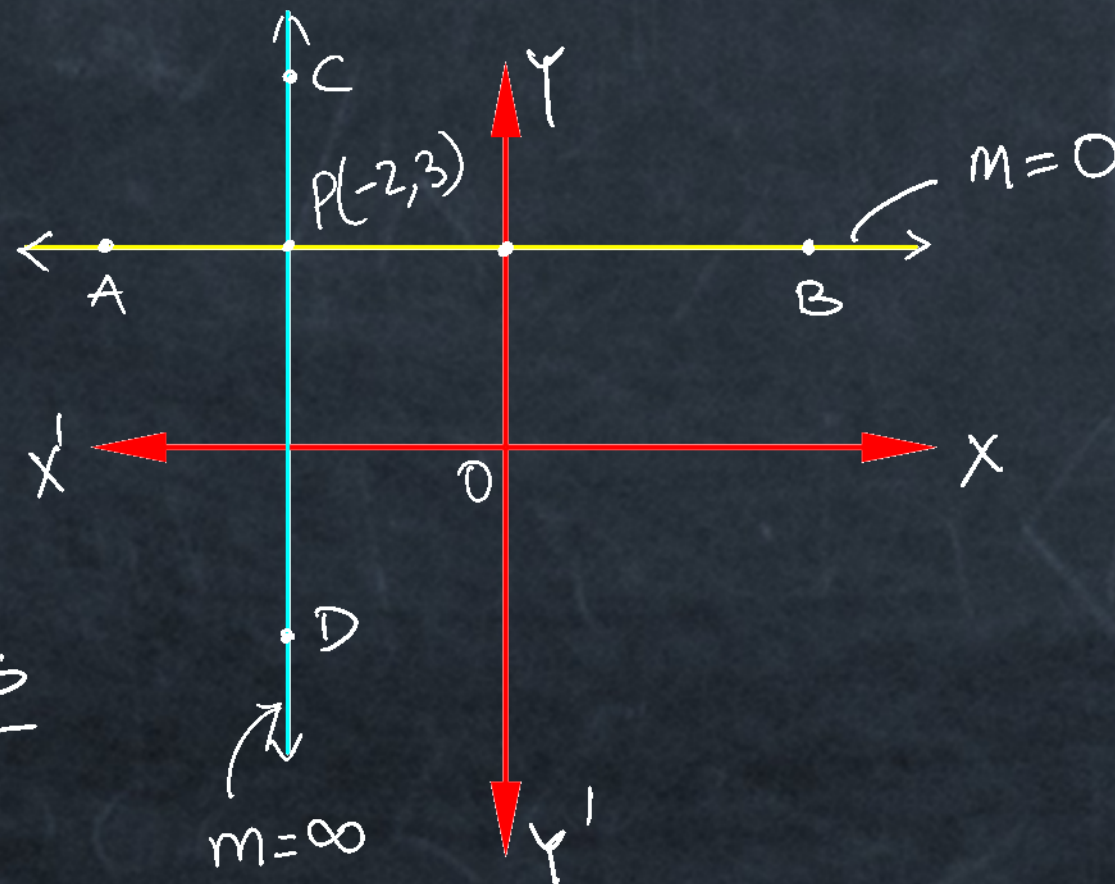
$$y \sin \omega - p \sin^2 \omega = -x \cos \omega + p^2 \cos^2 \omega$$

$$x \cos \omega + y \sin \omega = p(\sin^2 \omega + \cos^2 \omega)$$

$$\boxed{x \cos \omega + y \sin \omega = p}^*$$

Example 6 Find the equations of the lines parallel to axes and passing through $(-2, 3)$.

a) Parallel to X-axis



b) Parallel to Y-axis

Eqⁿ of AB // to X axis

$$y - 3 = 0(x + 2)$$

$$\boxed{y = 3}$$

Eqⁿ of CD // to Y axis

$$y - 3 = \infty(x + 2)$$

$$\left(\frac{y - 3}{\infty}\right) = x + 2$$

$$\boxed{x = -2}$$

Example 7 Find the equation of the line through $(-2, 3)$ with slope -4 .

$m = -4$, Line passes through $(-2, 3)$

Point-slope form: $y - y_1 = m(x - x_1)$

$$y - 3 = -4(x + 2)$$

$$y - 3 = -4x - 8$$

$$4x + y + 5 = 0$$

Example 8 Write the equation of the line through the points $(1, -1)$ and $(3, 5)$.

• Line passes through $(1, -1)$ & $(3, 5)$
 (x_1, y_1) (x_2, y_2)

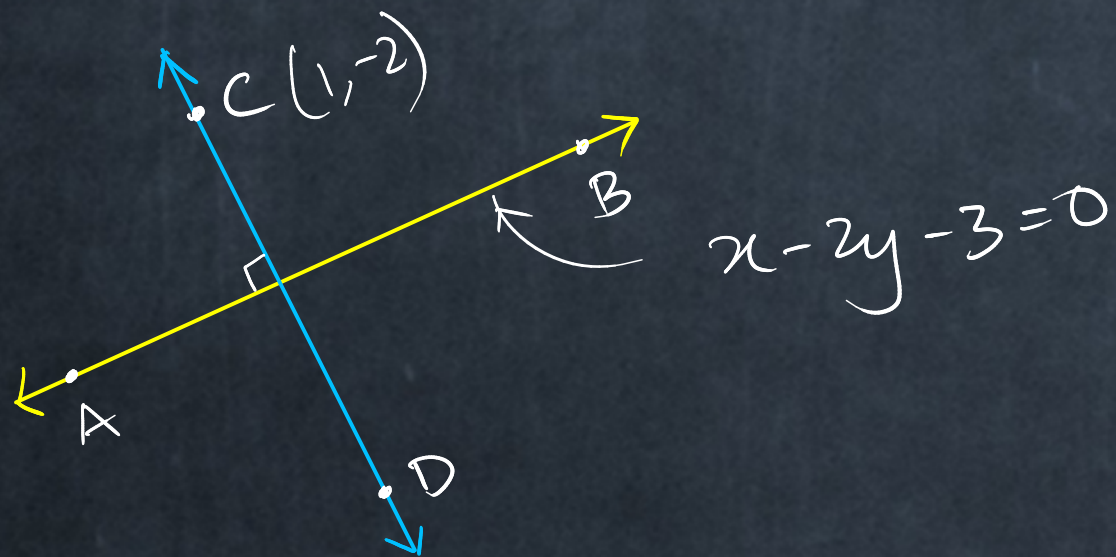
• Two pt. form: $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$

$$(y + 1) = \left(\frac{5 + 1}{3 - 1} \right) (x - 1)$$

$$y + 1 = 3(x - 1) \Rightarrow y + 1 = 3x - 3$$

$$3x - y - 4 = 0$$

Example 17 Find the equation of a line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point $(1, -2)$.



Slope of AB : $x - 2y - 3 = 0$
 $2y = x - 3$
 $y = \frac{1}{2}x - \frac{3}{2}$
 $m_{AB} = \frac{1}{2}$

$\Rightarrow m_{CD} = -2$ ($\because AB \perp CD$)

$\therefore$ Eqⁿ of CD with slope -2 , through $(1, -2)$

$$y + 2 = -2(x - 1)$$

$$y + 2 = -2x + 2$$

$$2x + y = 0$$

Example 18 Find the distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$.

Line: $3x - 4y - 26 = 0$

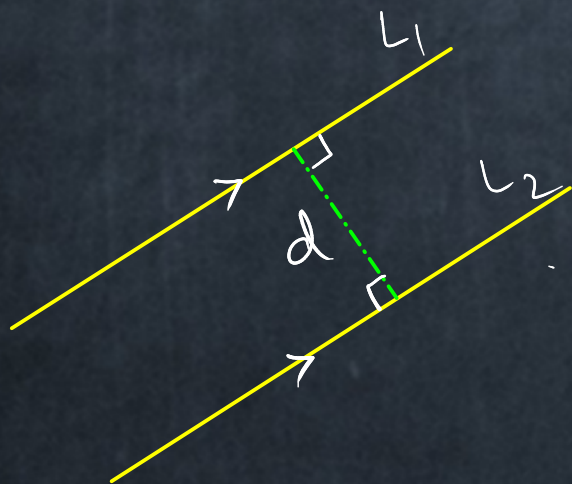
$A = 3, B = -4, C = -26$

$$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2} = 5$$

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

$$d = \left| \frac{3(3) + (-4)(-5) - 26}{5} \right| \Rightarrow d = \frac{3}{5} \text{ units}$$

Example 19 Find the distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$



$$3x - 4y + 7 = 0$$

$$A_1 = 3 \quad B_1 = -4 \quad C_1 = 7$$

$$3x - 4y + 5 = 0$$

$$A_2 = 3 \quad B_2 = -4 \quad C_2 = 5$$

$$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2} = 5$$

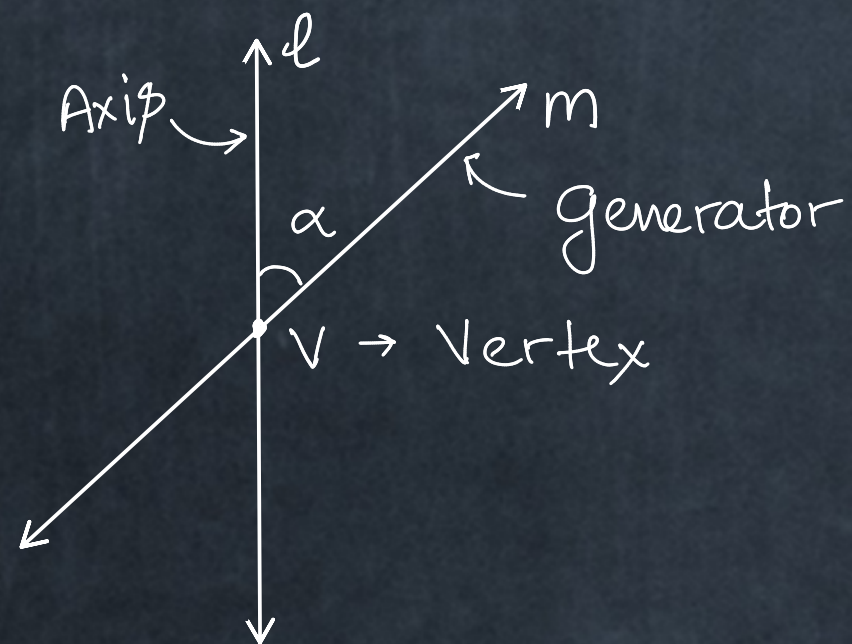
$$d = \left| \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} \right|$$

$$d = \left| \frac{5 - 7}{5} \right|$$

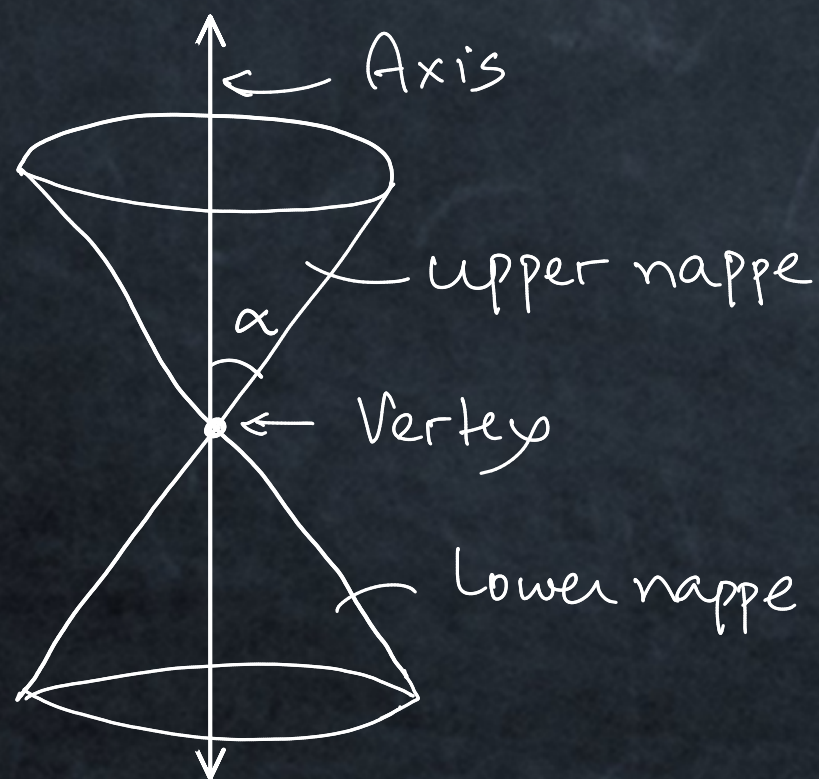
$$d = \frac{2}{5} \text{ units}$$

CONIC SECTIONS

CONIC SECTIONS (OR CONICS)

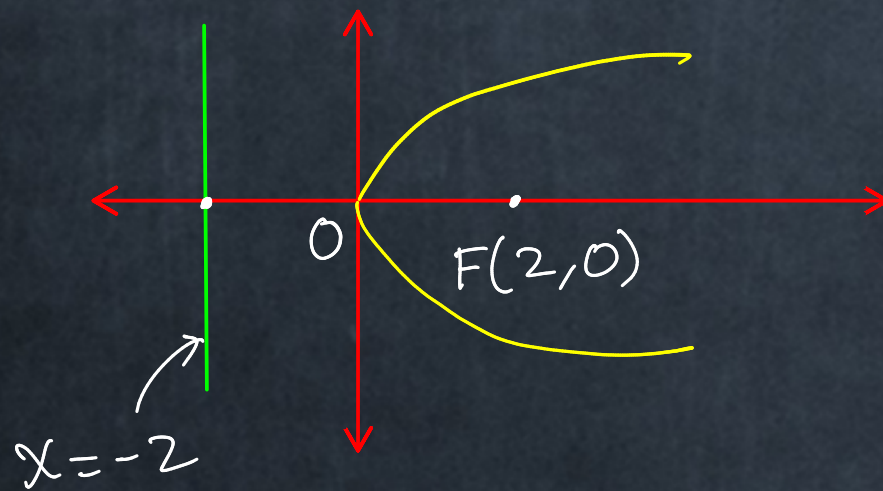


- As the generator 'm' rotates about vertex V, maintaining angle α , the surface formed above & below the vertex is a Double napped right circular cone or simply Cone.



- The section obtained by the intersection of a plane with the cone is termed a Conic Section
↳ "Curves"
- Types of curves depend on
 - Where the plane cuts
 - Angle at which it cuts
- Sections
 - Plane DOES NOT pass through Vertex
 - Plane passes through Vertex

Example 6 Find the equation of the parabola with focus $(2,0)$ and directrix $x = -2$.

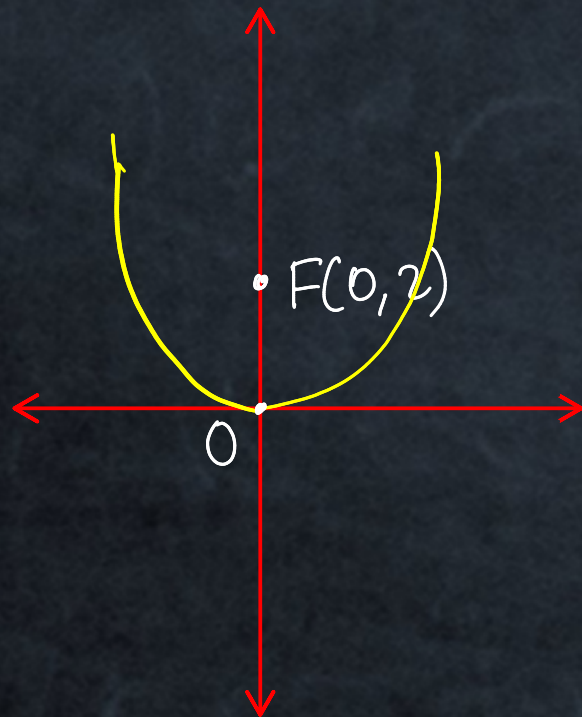


$$a = 2 \quad y^2 = 4ax$$

$$y^2 = 4(2)x$$

$$\text{Equation of parabola : } y^2 = 8x$$

Example 7 Find the equation of the parabola with vertex at $(0, 0)$ and focus at $(0, 2)$.



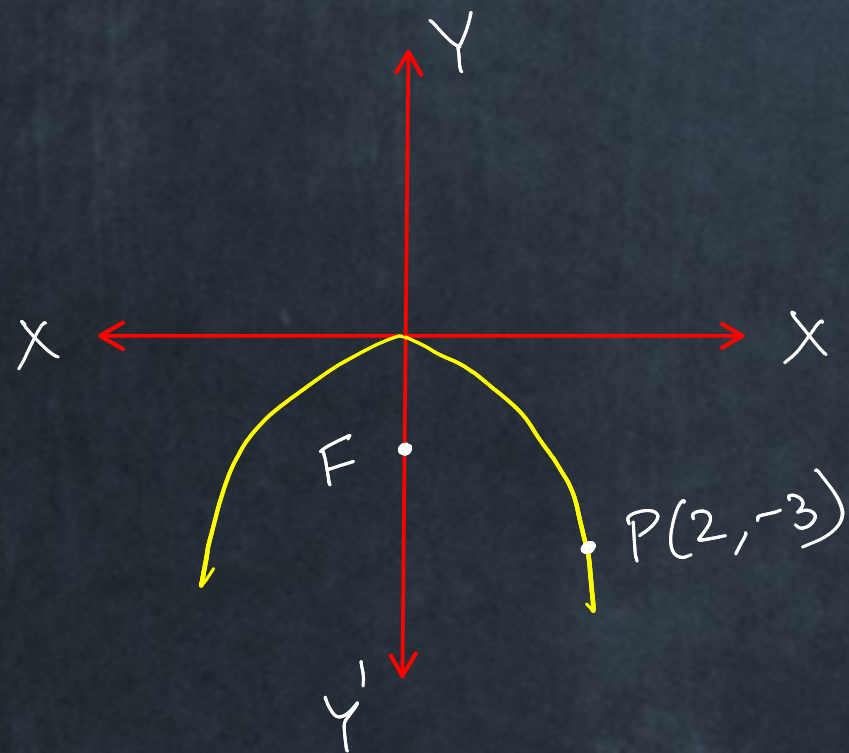
$$a = 2$$

$$x^2 = 4ay$$

$$x^2 = 4(2)y$$

$$\text{Eqn of parabola : } x^2 = 8y$$

Example 8 Find the equation of the parabola which is symmetric about the y-axis, and passes through the point $(2, -3)$.



Given: Axis of symmetry is Y-axis
 · Parabola passes through $(2, -3)$
 which lies in Quadrant IV.
 $\Rightarrow$ Parabola opens downward

$$x^2 = -4ay \quad \checkmark$$

Since $(2, -3)$ lies on parabola

$$(2)^2 = -4a(-3)$$

$$a = +\frac{1}{3}$$

$$\therefore \text{The Eq}^n \quad x^2 = -4\left(\frac{1}{3}\right)y$$

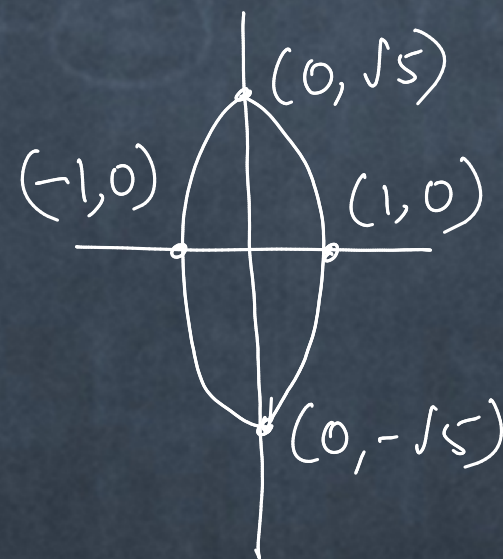
$$x^2 = -\frac{4}{3}y$$

14. Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

$$a = \sqrt{5} \quad b = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \Rightarrow$$

$$\frac{x^2}{1} + \frac{y^2}{5} = 1$$



15. Length of major axis 26, foci $(\pm 5, 0)$

$$\text{Given Length of major axis} = 2a = 26 \Rightarrow a = 13$$

$$c = 5$$

$$b = \sqrt{a^2 - c^2} = \sqrt{169 - 25} = 12$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow$$

$$\frac{x^2}{169} + \frac{y^2}{144} = 1$$

16. Length of minor axis 16, foci $(0, \pm 6)$.

$$\text{Length of minor axis} = 2b = 16 \Rightarrow b = 8$$

$$\text{foci} = (0, \pm 6) \Rightarrow c = 6$$

Also Vertical Ellipse

$$a = \sqrt{b^2 + c^2} = \sqrt{64 + 36} = 10$$

$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

17. Foci $(\pm 3, 0)$, $a = 4$

$\Rightarrow$ horizontal ellipse

$$c = 3, a = 4 \Rightarrow b = \sqrt{a^2 - c^2} = \sqrt{16 - 9} = \sqrt{7}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{16} + \frac{y^2}{7} = 1}$$

18. $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

$$b = 3, c = 4 \Rightarrow a = \sqrt{b^2 + c^2} = \sqrt{9 + 16} = 5$$

Horizontal Ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

INTRODUCTION TO 3D GEOMETRY

Example 1 In Fig 12.3, if P is (2,4,5), find the coordinates of F.

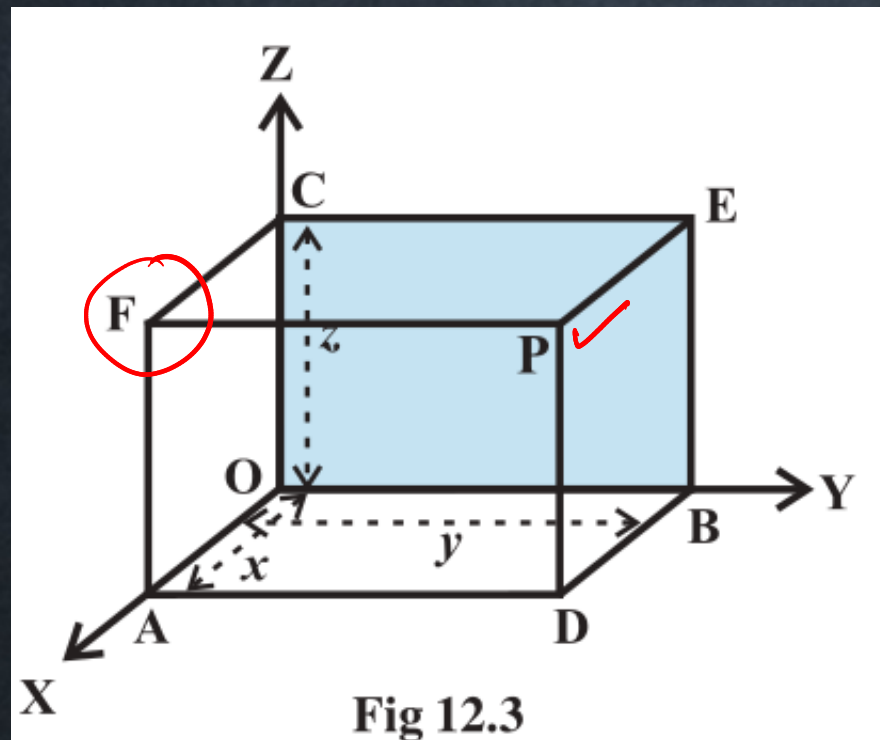
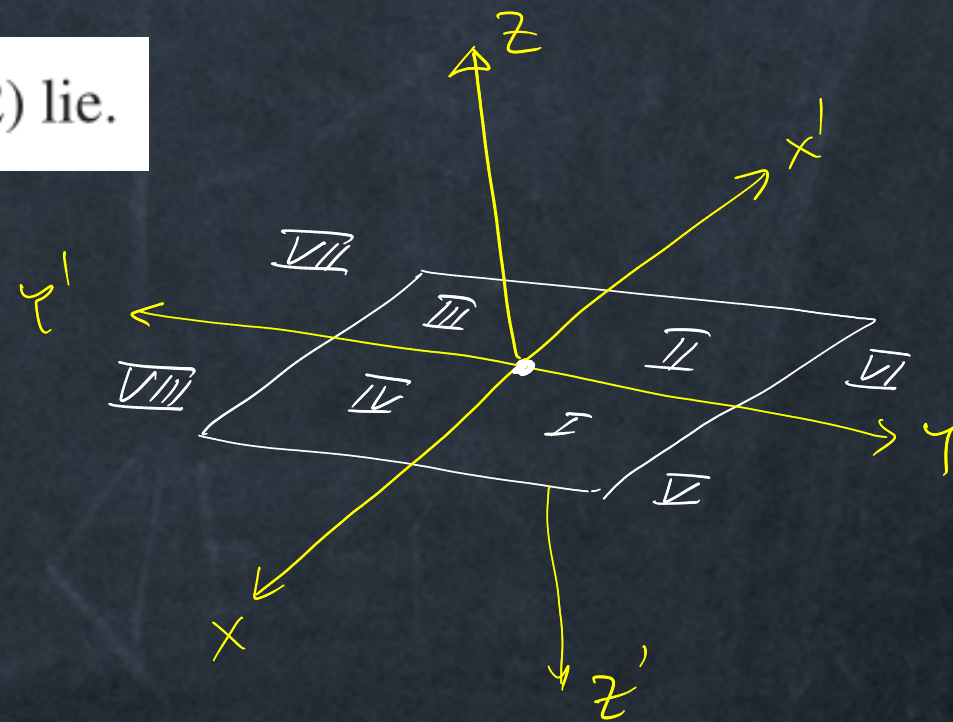


Fig 12.3

$$F(2, 0, 5)$$

Example 2 Find the octant in which the points $(-3, 1, 2)$ and $(-3, 1, -2)$ lie.

$$\begin{array}{cc} P(-3, 1, 2) & Q(-3, 1, -2) \\ \Downarrow & \Downarrow \\ \text{II}^{\text{nd}} \text{ Octant} & \text{VI}^{\text{th}} \text{ Octant} \end{array}$$



EXERCISE 12.2

1. Find the distance between the following pairs of points:

- | | |
|----------------------------------|---------------------------------|
| (i) (2, 3, 5) and (4, 3, 1) | (ii) (-3, 7, 2) and (2, 4, -1) |
| (iii) (-1, 3, -4) and (1, -3, 4) | (iv) (2, -1, 3) and (-2, 1, 3). |

(i) $A(2, 3, 5)$, $B(4, 3, 1)$; $AB = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} = \sqrt{4+0+16} = \sqrt{20} = 2\sqrt{5} \text{ units}$

(ii) $A(-3, 7, 2)$, $B(2, 4, -1)$ $AB = \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} = \sqrt{25+9+9} = \sqrt{43} \text{ units}$

(iii) $A(-1, 3, -4)$, $B(1, -3, 4)$ $AB = \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2} = \sqrt{4+36+64} = \sqrt{104}$
 $= 2\sqrt{26} \text{ units}$
 $\begin{array}{r} 2 \overline{)104} \\ \underline{4} \\ 64 \\ \underline{64} \\ 0 \end{array}$

(iv) $A(2, -1, 3)$, $B(-2, 1, 3)$ $AB = \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$
 $AB = \sqrt{16+4+0} = \sqrt{20} = 2\sqrt{5} \text{ units}$

2. Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.

$A(-2, 3, 5)$ $B(1, 2, 3)$ $C(7, 0, -1)$

$AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14} \text{ units}$

$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14} \text{ units}$

$AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14} \text{ units}$

Since $AB + BC = AC \Rightarrow A, B \& C$ are collinear

3. Verify the following:

- (i) $(0, 7, -10)$, $(1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.
- (ii) $(0, 7, 10)$, $(-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right angled triangle.
- (iii) $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

(i) $A(0, 7, -10)$ $B(1, 6, -6)$ $C(4, 9, -6)$

$$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2} = \sqrt{16+4+16} = \sqrt{36} = 6 \text{ units}$$

Since $AB = BC \therefore \triangle ABC$ is Isosceles

(ii) $A(0, 7, 10)$ $B(-1, 6, 6)$ $C(-4, 9, 6)$

$$AB^2 = (-1-0)^2 + (6-7)^2 + (6-10)^2 = 1+1+16 = 18$$

$$BC^2 = (-4+1)^2 + (9-6)^2 + (6-6)^2 = 9+9+0 = 18$$

$$AC^2 = (-4-0)^2 + (9-7)^2 + (6-10)^2 = 16+4+16 = 36$$

Since $AB^2 + BC^2 = AC^2$ Here the vertices form a right triangle

2. Given that P (3, 2, -4), Q (5, 4, -6) and R (9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Given P(3, 2, -4) Q(5, 4, -6) & R(9, 8, -10) are collinear

Let Q divide PR in the ratio $k:1$

$$\text{Coord. of Q} \left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1} \right)$$

Compare Q (5 , 4 , -6)

$$\frac{9k+3}{k+1} = 5$$

$$9k+3 = 5k+5$$

$$4k = 2$$

$$k = \frac{1}{2}$$

$$\frac{8k+2}{k+1} = 4$$

$$8k+2 = 4k+4$$

$$4k = 2$$

$$k = \frac{1}{2}$$

$$\frac{-10k-4}{k+1} = -6$$

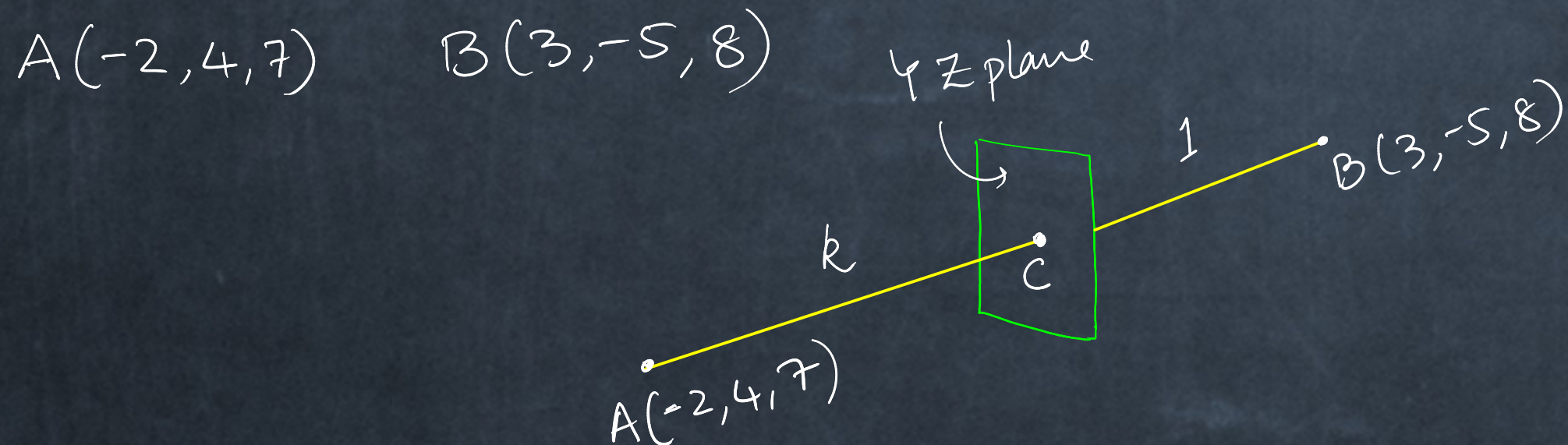
$$-10k-4 = -6k-6$$

$$4k = 2$$

$$k = \frac{1}{2}$$

$\therefore$ Q divides PR internally in the ratio 1:2

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$.



Let C on plane YZ divide line AB in the ratio $k:1$

By section formula Coords. of C $\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1} \right)$

We know for any pt. on the YZ plane, its x-coord. is zero

$$\frac{3k-2}{k+1} = 0 \Rightarrow k = \frac{2}{3}$$

YZ plane divides AB internally in the ratio 2:3