

UNIT I

SETS AND FUNCTIONS

1. Sets
2. Relations & Functions
3. Trigonometric Functions

SETS

Definition of Set: A set is a collection of well defined objects.

• Some Examples of collection of :

- (i) Odd natural numbers less than 10, i.e., 1, 3, 5, 7, 9
- (ii) The rivers of India
- (iii) The vowels in the English alphabet, namely, *a, e, i, o, u*
- (iv) Various kinds of triangles
- (v) Prime factors of 210, namely, 2, 3, 5 and 7
- (vi) The solution of the equation: $x^2 - 5x + 6 = 0$, viz, 2 and 3.

We note that each of the above example is a well-defined collection of objects in

These are a collection
of WELL DEFINED
objects.
Hence are sets

• Some Imp. Number sets

N : Set of all natural numbers
 W : Set of all whole numbers
 Q : Set of all rational numbers.
 R : Set of all real numbers.
 Z^+ : Set of all the Integers.

$p, q \in Z$
 $\frac{p}{q}$, $q \neq 0$ ✓ Coprimes
 $HCF(p, q) = 1$

Subsets of Real Numbers

Standard Subsets
Non standard Subsets

Standard Subsets of $\mathbb{R}$

$$\mathbb{N} = \{1, 2, 3, \dots\} \Rightarrow \mathbb{N} \subset \mathbb{R}$$

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\} \Rightarrow \mathbb{Z} \subset \mathbb{R}$$

$$\mathbb{Q} = \{x: x = \frac{p}{q}, p, q \in \mathbb{Z}, q \neq 0\} \Rightarrow \mathbb{Q} \subset \mathbb{R}$$

*

Irrational
Nos

$$\mathbb{T} = \{x: x \in \mathbb{R}, x \notin \mathbb{Q}\}$$

$$\Rightarrow \mathbb{T} \subset \mathbb{R}$$

Some of the obvious relations among these subsets are:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}, \mathbb{Q} \subset \mathbb{R}, \mathbb{T} \subset \mathbb{R}, \mathbb{N} \not\subset \mathbb{T}.$$

Non standard Subsets of $\mathbb{R}$ (or) Intervals as subsets of $\mathbb{R}$

- Consider the interval of real Nos between a & b where $a < b$

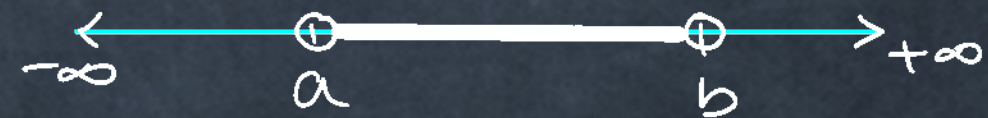
Finite Interval form

Set Builder form

Numberline representation

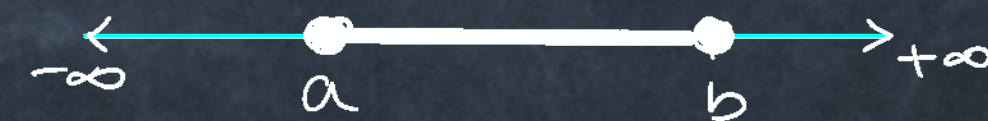
$$(a, b)$$

$$\{x: x \in \mathbb{R}, a < x < b\}$$



$$[a, b]$$

$$\{x: x \in \mathbb{R}, a \leq x \leq b\}$$



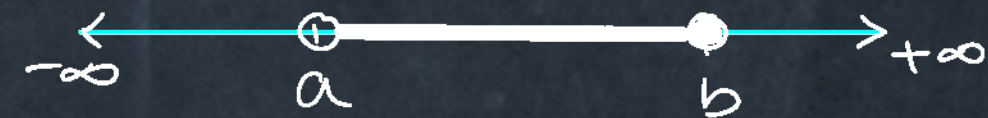
$$[a, b)$$

$$\{x: x \in \mathbb{R}, a \leq x < b\}$$



$$(a, b]$$

$$\{x: x \in \mathbb{R}, a < x \leq b\}$$



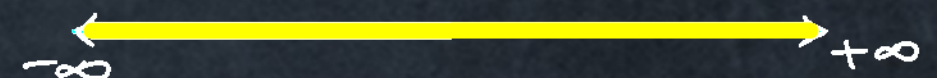
Infinite Interval form

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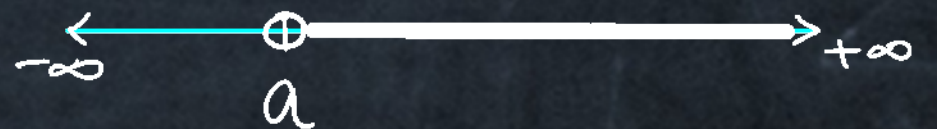
$$(-\infty, \infty)$$

$$\{x: x \in \mathbb{R}\} \text{ or } \{x: -\infty < x < \infty\}$$



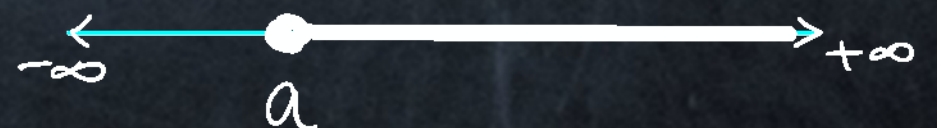
$$(a, \infty)$$

$$\{x: x \in \mathbb{R}, x > a\}$$



$$[a, \infty)$$

$$\{x: x \in \mathbb{R}, x \geq a\}$$



$$(-\infty, a)$$

$$\{x: x \in \mathbb{R}, x < a\}$$



$$(-\infty, a]$$

$$\{x: x \in \mathbb{R}, x \leq a\}$$



4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find
- | | | | |
|-----------------------|------------------------|-------------------------|-----------------|
| (i) $A \cup B$ | (ii) $A \cup C$ | (iii) $B \cup C$ | (iv) $B \cup D$ |
| (v) $A \cup B \cup C$ | (vi) $A \cup B \cup D$ | (vii) $B \cup C \cup D$ | |

$$A = \{1, 2, 3, 4\} \quad B = \{3, 4, 5, 6\} \quad C = \{5, 6, 7, 8\} \quad D = \{7, 8, 9, 10\}$$

$$(i) A \cup B = \{1, 2, 3, 4, 5, 6\} \checkmark$$

$$(ii) A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$(iii) B \cup C = \{3, 4, 5, 6, 7, 8\}$$

$$(iv) B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$(v) A \cup B \cup C = (A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$(vi) A \cup B \cup D = A \cup (B \cup D) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$(vii) B \cup C \cup D = (B \cup C) \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

5. Find the intersection of each pair of sets of question 1 above.

- (i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$
(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$
(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$
 $B = \{x : x \text{ is a natural number less than } 6\}$
(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$
 $B = \{x : x \text{ is a natural number and } 6 < x < 10\}$
(v) $A = \{1, 2, 3\}$, $B = \phi$

$$(i) \quad X \cap Y = \{1, 3\}$$

$$(iii) \quad A = \{3, 6, 9, \dots\}$$

$$(ii) \quad A \cap B = \{a\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$A \cap B = \{3\}$$

$$(iv) \quad A = \{2, 3, 4, 5, 6\}$$

$$B = \{7, 8, 9\}$$

$$A \cap B = \phi$$

$$(v) \quad A = \{1, 2, 3\} \quad B = \phi$$

$$A \cap \phi = \phi$$

RELATIONS & FUNCTIONS

Cartesian Product of Sets

Ordered pair - A pair of elements in a specified order.

- Also known as 2-tuple
- Eg: $(5, 6)$, $(-1, 7)$ etc. -
- If A & B are two sets, then ordered pair (a, b) means $a \in A, b \in B$.

NOTE: $(a, b) \neq (b, a)$ unless $a = b$

- A 3-tuple or Ordered triplet is a specified arrangement of 3 elements.
Eg: $(1, 2, -4)$ etc.
- Two ordered pairs are equal if their corresponding elements are equal

Eg: $(x+3, y-1) = (7, 8)$

$$\begin{aligned} x+3 &= 7 \Rightarrow x = 4 \\ y-1 &= 8 \Rightarrow y = 9 \end{aligned}$$

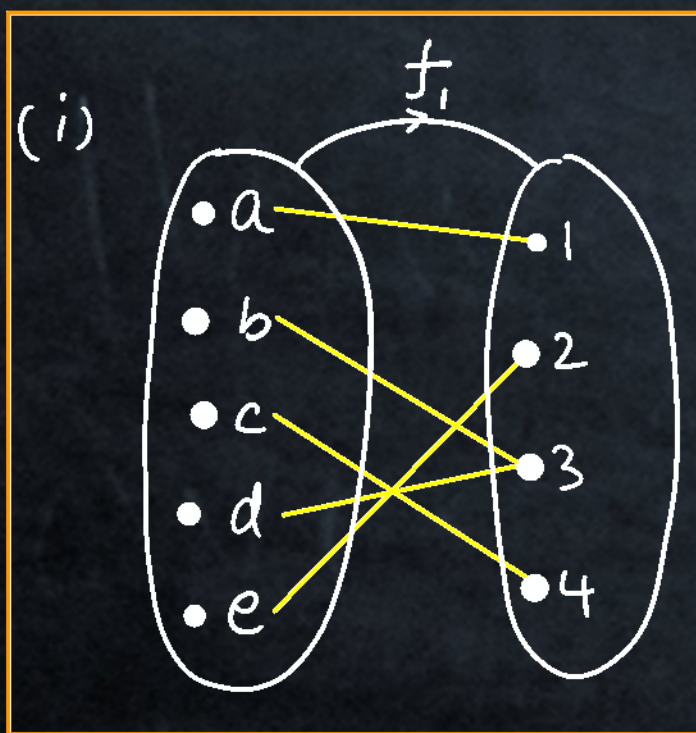
FUNCTIONS → (Also known as Maps or Mappings)

Definition # 1 - A relation f from set A to set B (both non-empty), is said to be a function if every element of set A (Domain) has one & only one (unique) image in set B (Co-Domain)

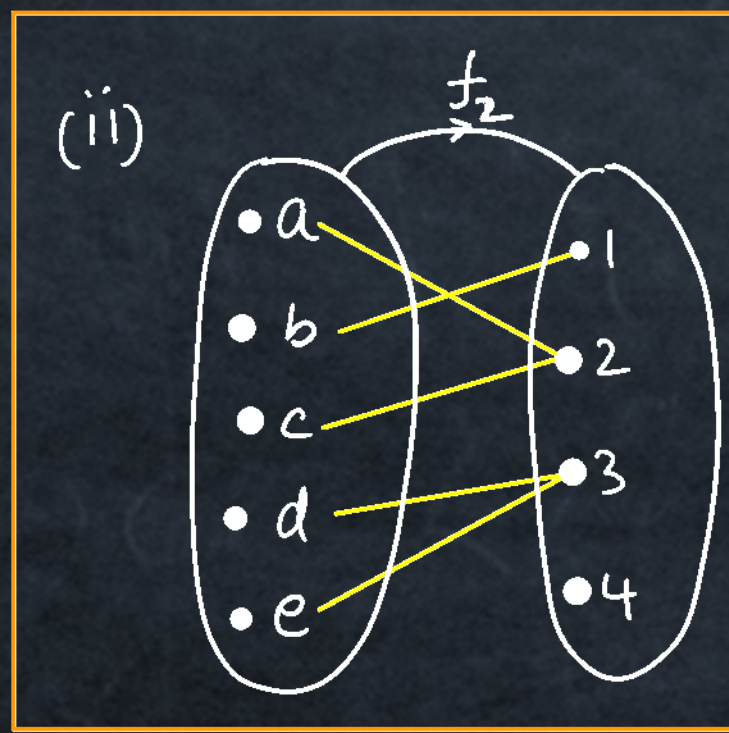
(OR)

A function is a relation in which every element of Domain has a unique image in Co-Domain.

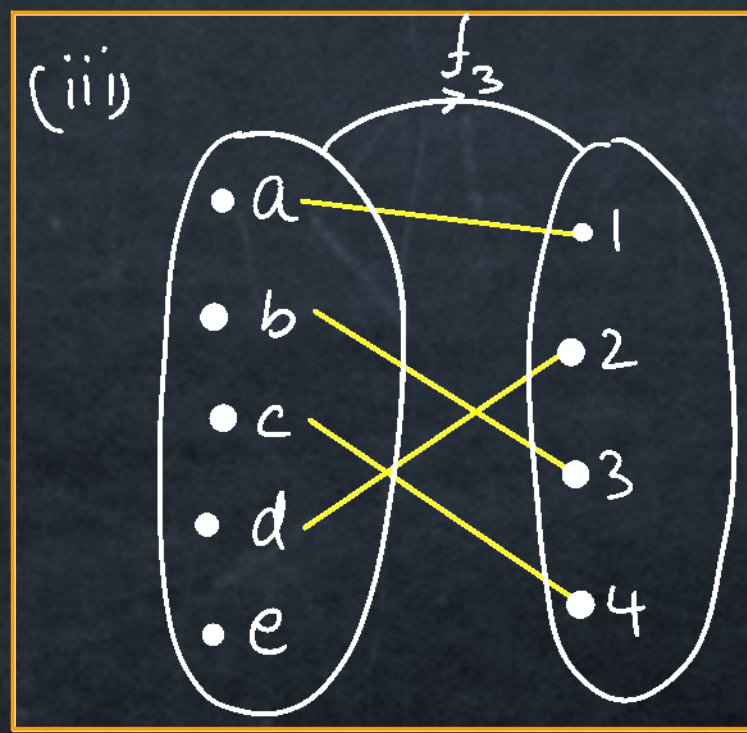
Ex: Identify which are functions & justify your answer



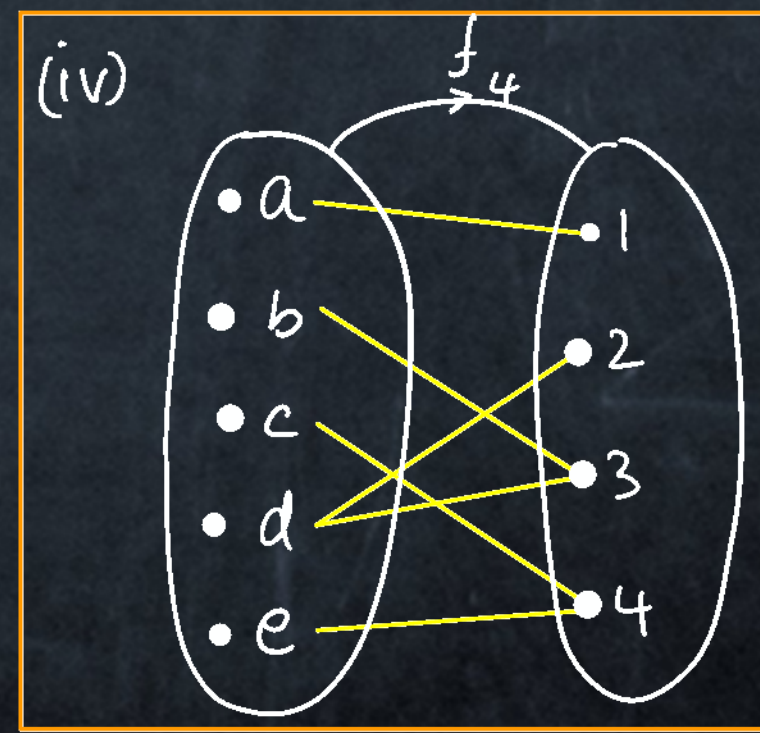
f_1 is a function



f_2 is a function



f_3 is NOT a function
∵ 'e' has no image



f_4 is NOT a function
∵ 'd' has 2 images.

Definition #2 : A function f is a relation from set A to set B (both non-empty) such that the domain of f is set A and no two distinct ordered pairs in f , have the same first element.

Ex : Check whether the following relations are functions. Justify

$$A = \{1, 2, 3, 4\} \quad B = \{1, 5, 9, 11, 15\}$$

$$(i) \quad f_1 = \{(1, 5), (\hat{2}, 9), (3, 1), (4, 5), (\hat{2}, 11)\}$$

Not a function since 2 is repeated.

$$(ii) \quad f_2 = \{(1, 5), (2, 9), (3, 1)\}$$

$$\text{Domain} = \{1, 2, 3\} \neq A$$

Not a function since Domain $\neq A$.

NOTE

1. If f is a function from A to B and $(a, b) \in f$, then $f(a) = b$, where b is Image of a under f & a is pre-Image of b under f .

2. Denoted by $f : A \rightarrow B$

③ Multiply or Divide by any -ve number, sign of inequality WILL CHANGE.

$$5 > 2$$

$$(-1) 5 < (-1) 2$$

$$5 > 2$$

$$\frac{5}{(-1)} < \frac{2}{(-1)}$$

④ Reciprocal Rule:

If given $a < b$, we need left limit also say x

$$\begin{array}{c} a < b \\ \oplus \quad \oplus \\ x < a < b \end{array}$$

$$\frac{1}{x} > \frac{1}{a} > \frac{1}{b}$$

$$\begin{array}{c} a < b \\ \ominus \quad \ominus \\ x < a < b \end{array}$$

$$\frac{1}{x} > \frac{1}{a} > \frac{1}{b}$$

$$\begin{array}{c} a < b \\ \oplus \quad \ominus \\ x < a < b \end{array}$$

⇓
DO NOT TAKE
RECIPROCAL

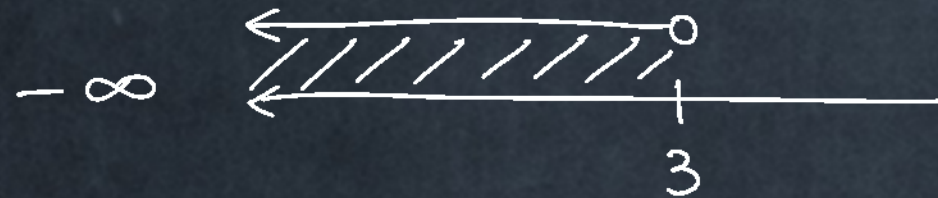
⇓
Analyse

$$(x, 0) \cup (0, b)$$

Linear Inequality

Ex 1 Solve : $3x - 7 < 2$

$$3x - \cancel{7} + \cancel{7} < 2 + 7 \Rightarrow 3x < 9 \Rightarrow \frac{3x}{3} < \frac{9}{3} \Rightarrow x < 3$$



$$x \in (-\infty, 3)$$

Ex 2 : $2 < 3 - 4x \leq 10$

$$2 < 3 - 4x$$

$$4x < 3 - 2$$

$$x < \frac{1}{4}$$

$$3 - 4x \leq 10$$

$$3 - 10 \leq 4x$$

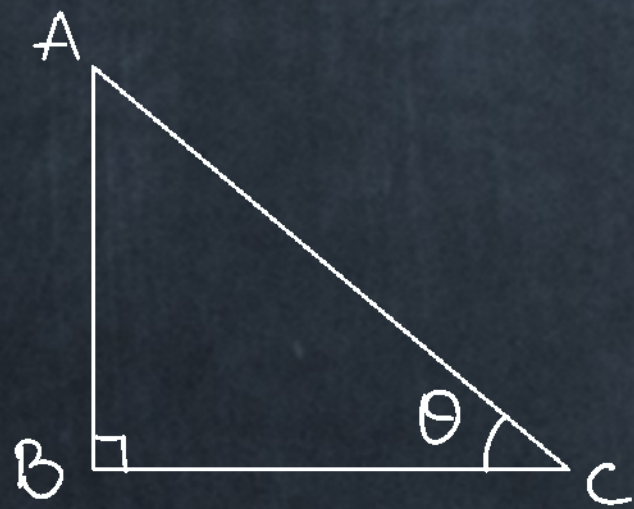
$$-\frac{7}{4} \leq x \text{ or } x \geq -\frac{7}{4}$$

$$-\frac{7}{4} \leq x < \frac{1}{4}$$

$$x \in \left[-\frac{7}{4}, \frac{1}{4}\right)$$

TRIGONOMETRIC FUNCTIONS

Recap of class x Trig.



$$\sin \theta = \frac{\text{OPP}}{\text{hyp}} = \frac{AB}{AC}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{BC}{AC}$$

$$\tan \theta = \frac{\text{OPP}}{\text{adj}} = \frac{AB}{BC}$$

$$\operatorname{Cosec} \theta = \frac{1}{\sin \theta} = \frac{AC}{AB}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{AC}{BC}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{BC}{AB}$$

T. ratios for Standard angles

	0°	30°	45°	60°	90°
sin	0	1/2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	N.D.
Cosec	N.D.	2	√2	2/√3	1
Sec	1	2/√3	√2	2	N.D.
Cot	N.D.	√3	1	1/√3	0

Note:

$$\frac{0}{1} = 0$$

$$\frac{1}{0} = \text{ND}$$

$$\frac{1}{\text{ND}} = 0$$

$$\text{ND} = +\infty$$

Trig. Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\operatorname{Cosec}^2 \theta - \cot^2 \theta = 1$$

$$\rightarrow \sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta}$$

$$\rightarrow \operatorname{Cosec} \theta - \cot \theta = \frac{1}{\operatorname{Cosec} \theta + \cot \theta}$$

Example 7 If $\cot x = -\frac{5}{12}$, x lies in second quadrant, find the values of other five trigonometric functions.

Solu: $\cot x = -\frac{5}{12}$ $\tan x = -\frac{12}{5}$

From $\operatorname{cosec}^2 x - \cot^2 x = 1$

$$\operatorname{cosec}^2 x - \left(-\frac{5}{12}\right)^2 = 1 \Rightarrow \operatorname{cosec}^2 x - \frac{25}{144} = 1 \Rightarrow \operatorname{cosec}^2 x = 1 + \frac{25}{144}$$

$$\Rightarrow \operatorname{cosec}^2 x = \frac{169}{144} \Rightarrow \operatorname{cosec} x = \pm \frac{13}{12}$$

Since x lies in QII, $\operatorname{cosec} x = +\frac{13}{12}$, $\sin x = \frac{12}{13}$

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow -\frac{12}{5} = \frac{12/13}{\cos x} \Rightarrow \cos x = \frac{12}{13} \times \frac{-5}{12} = -\frac{5}{13}$$

$$\sec x = -\frac{13}{5}$$

$$\sin x = \frac{12}{13}, \cos x = -\frac{5}{13}, \tan x = -\frac{12}{5}$$

$$\operatorname{cosec} x = \frac{13}{12}, \sec x = -\frac{13}{5}, \cot x = -\frac{5}{12}$$

Example 8 Find the value of $\sin \frac{31\pi}{3}$.

Approach #1

$$\sin \left(\frac{31 \times 180}{3} \right) = \sin (1860)$$

• Quadrant & Sign

QI $\Rightarrow \sin x$ (+ve)

$$\begin{aligned} 1860 &= 90n + \theta \\ &= 90 \times 20 + 60 \\ &\quad \text{even} \\ &= + \sin 60^\circ = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\sin \frac{31\pi}{3} = \frac{\sqrt{3}}{2}$$

Approach #2

$$\begin{aligned} \sin \left(\frac{31\pi}{3} \right) &= \sin \left(10\pi + \frac{\pi}{3} \right) \\ \sin(2n\pi + x) &= \sin x \\ \sin \left[5(2\pi) + \frac{\pi}{3} \right] &= \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\sin \left(\frac{31\pi}{3} \right) = \frac{\sqrt{3}}{2}$$

$$\begin{array}{r} 31 \\ 6 \\ \hline 1860 \end{array}$$

$$\begin{array}{r} 1860 \\ 360 \\ 5 \\ \hline 1800 \end{array}$$

$$\begin{array}{r} 3 \overline{)31} \\ \underline{30} \\ 1 \end{array}$$

$$31 = 10 + \frac{1}{3}$$

Prove the following:

6. $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

$$\text{LHS} = \underbrace{\cos\left(\frac{\pi}{4} - x\right)}_A \underbrace{\cos\left(\frac{\pi}{4} - y\right)}_B - \underbrace{\sin\left(\frac{\pi}{4} - x\right)}_A \underbrace{\sin\left(\frac{\pi}{4} - y\right)}_B$$



$$\cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$\Rightarrow \cos\left[\frac{\pi}{4} - x + \frac{\pi}{4} - y\right] = \cos\left(\frac{\pi}{2} - (x+y)\right) = +\sin(x+y)$$

= RHS

Hence proved

7. $\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$

$$\begin{aligned} \text{LHS} &= \frac{\frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x}}{\frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x}} = \frac{1 + \tan x}{1 - \tan x} \times \frac{1 + \tan x}{1 - \tan x} \\ &= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{RHS} \checkmark \end{aligned}$$

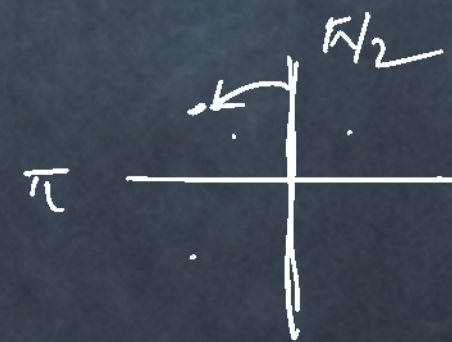
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

8. $\frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

$$\text{LHS} = \frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \frac{-\cos x \cos x}{\sin x (-\sin x)} = \frac{-\cos^2 x}{-\sin^2 x} = \cot^2 x = \text{RHS}$$

Hence proved



9. $\cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right] = 1$

$$\text{LHS} = \cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right]$$

$$= (+\sin x) (\cos x) \left[(+\tan x) + (+\cot x) \right]$$

$$= \sin x \cos x \left[\tan x + \cot x \right]$$

$$= \sin x \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right] = \cancel{\sin x \cos x} \left[\frac{\sin^2 x + \cos^2 x}{\cancel{\sin x \cos x}} \right] = 1$$

= RHS

Hence proved

