Similarity of Triangles

- Two triangles are said to similar iff
  - (i) their corresponding sides are proportional

    &
  - (ii) their corresponding angles are equal.

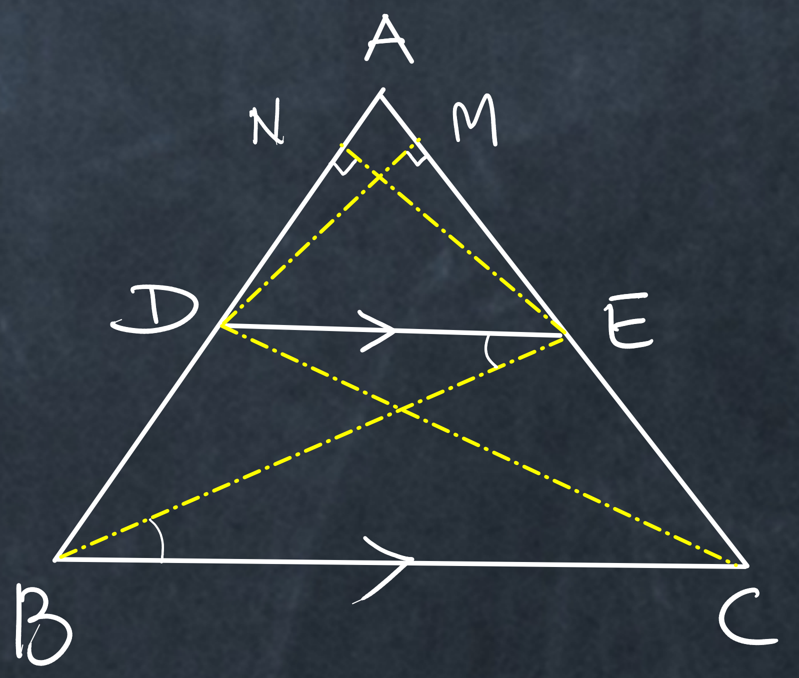
Theorem #1. Basic Proportionality Theorem or Thales' theorem (BPT.)

Theorem #2. Converse of BPT

joel samuel
9182099001

Basic Proportionality Theorem (BPT) or Thales theorem

Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio



Given: $DE \parallel BC$

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Altitude DM

Altitude EN

Join BE, DC

Proof:

$$\frac{ar(\triangle ADE)}{ar(\triangle DEB)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \text{— (i)}$$

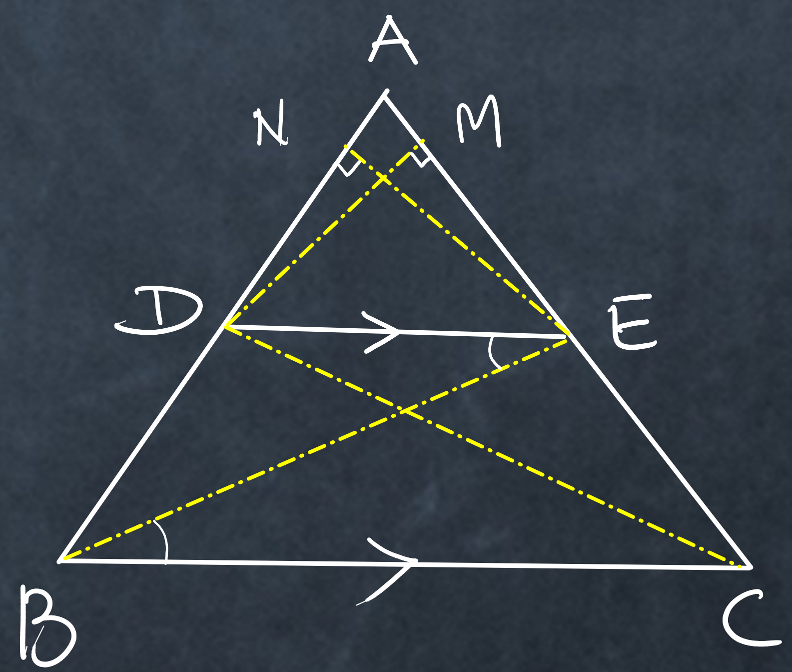
$$\frac{ar(\triangle ADE)}{ar(\triangle DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{— (ii)}$$

$ar(\triangle DEB) = ar(\triangle DEC)$ since they have the same base DE & lie between parallel line DE & BC

LHS of (i) = LHS of (ii) $\Rightarrow$ RHS of (i) = RHS of (ii)

joel samuel
9182099001

$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$ Hence proved.



Variant 2 of BPT

We know $\frac{AD}{DB} = \frac{AE}{EC}$

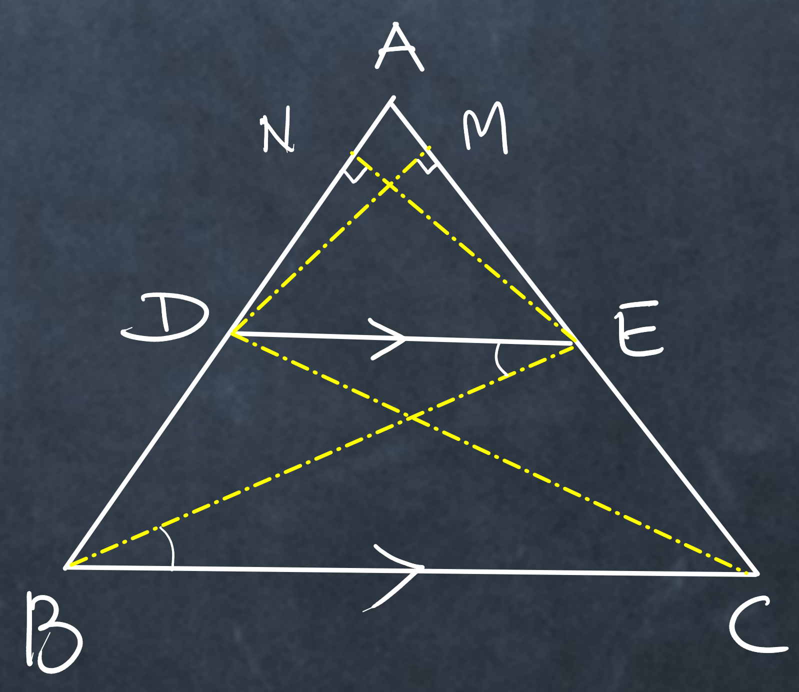
Take reciprocal both sides $\frac{DB}{AD} = \frac{EC}{AE}$

Add 1 both sides $\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$$\frac{DB + AD}{AD} = \frac{EC + AE}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

Take reciprocal $\boxed{\frac{AD}{AB} = \frac{AE}{AC}}$

Variant 3 of BPT



We know $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Add 1 both sides $\dfrac{AD}{DB} + 1 = \dfrac{AE}{EC} + 1$

$$\frac{AD+DB}{DB} = \frac{AE+EC}{EC}$$

$$\frac{AB}{DB} = \frac{AC}{EC}$$

Take reciprocal both sides $\boxed{\dfrac{DB}{AB} = \dfrac{EC}{AC}}$

NOTE: $\boxed{\dfrac{AD}{DB} = \dfrac{AE}{EC}}$ $\boxed{\dfrac{AD}{AB} = \dfrac{AE}{AC}}$ $\boxed{\dfrac{DB}{AB} = \dfrac{EC}{AC}}$

all three forms represent B.P.T.

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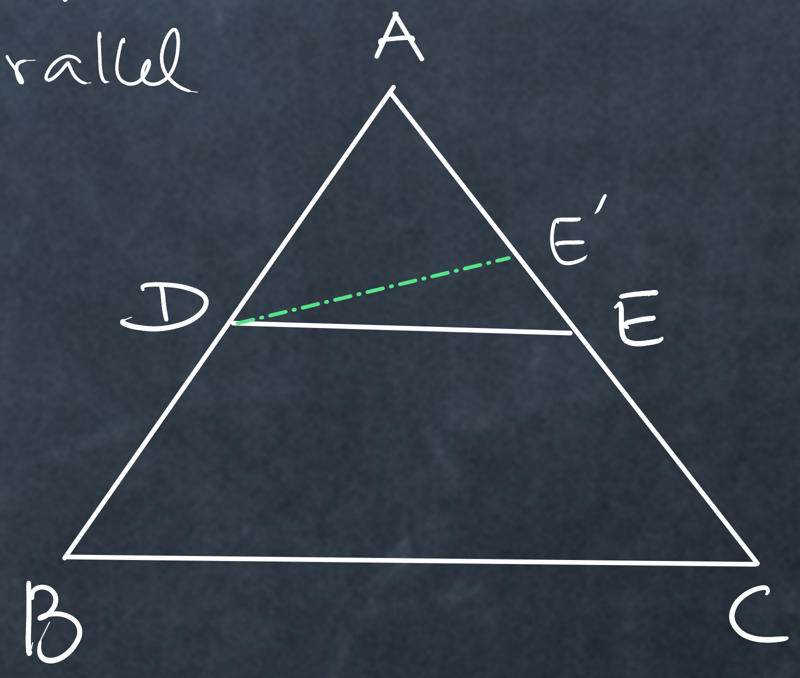
Converse of BPT

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side



Given: $\frac{AD}{DB} = \frac{AE}{EC}$ —(i) To prove $DE \parallel BC$

Construction: Draw DE'

Proof: Assume DE' is parallel to BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE'}{E'C} \text{ —(ii)} \quad (\because \text{BPT})$$

From (i) & (ii) $\frac{AE}{EC} = \frac{AE'}{E'C}$

Add 1 both sides $\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1 \Rightarrow \frac{AE+EC}{EC} = \frac{AE'+E'C}{E'C} \Rightarrow \frac{\cancel{AC}}{EC} = \frac{\cancel{AC}}{E'C}$

$$\Rightarrow EC = E'C$$

$\Rightarrow \boxed{DE \parallel BC}$ ($\because$ E' & E must be the same point)

Hence proved

joel samuel
9182099001

Example 2 : ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB (see Fig. 6.14). Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

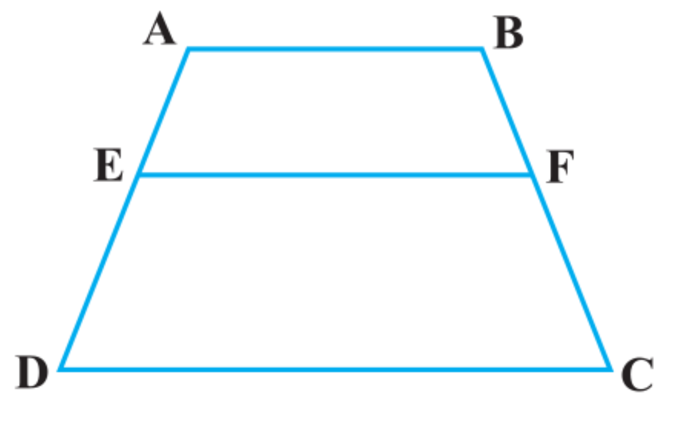
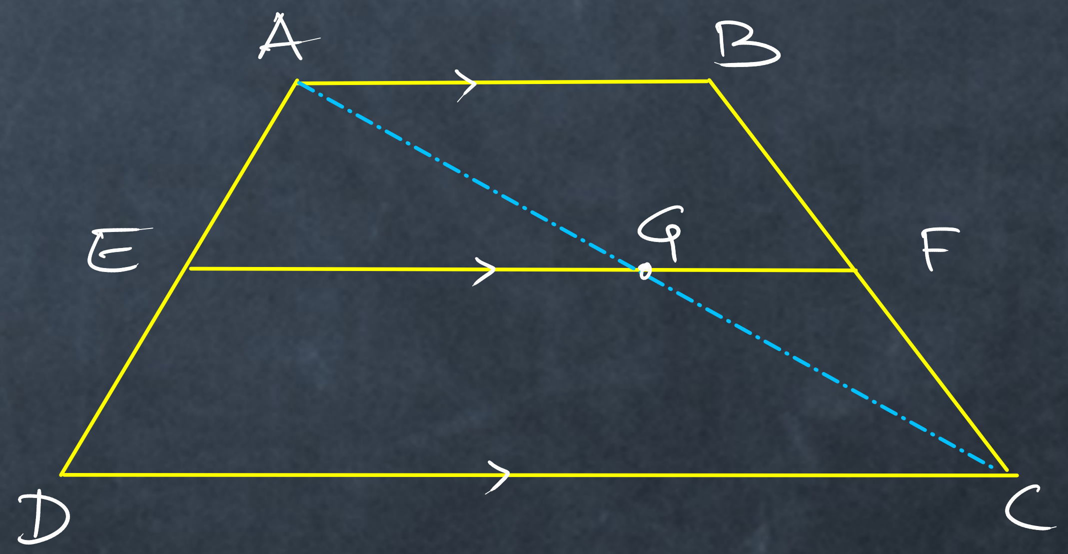


Fig. 6.14



Given: • AB // DC // EF

Construction: Join AC

To show: $\frac{AE}{ED} = \frac{BF}{FC}$

Proof: Consider $\triangle ADC$

$$\frac{AE}{ED} = \frac{AG}{GC} \quad \text{—(i)} \quad (\because \text{BPT})$$

Consider $\triangle CAB$

$$\frac{FC}{BF} = \frac{GC}{AG} \quad \text{or} \quad \frac{BF}{FC} = \frac{AG}{GC} \quad \text{—(ii)} \quad (\because \text{Taking reciprocals})$$

From (i) & (ii) $\boxed{\frac{AE}{ED} = \frac{BF}{FC}}$ Hence proved

Example 3 : In Fig. 6.16, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

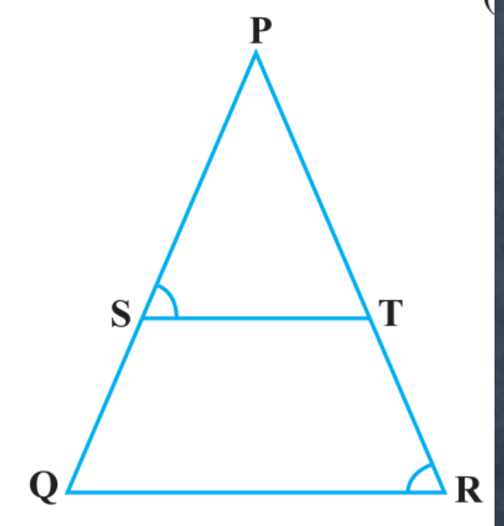
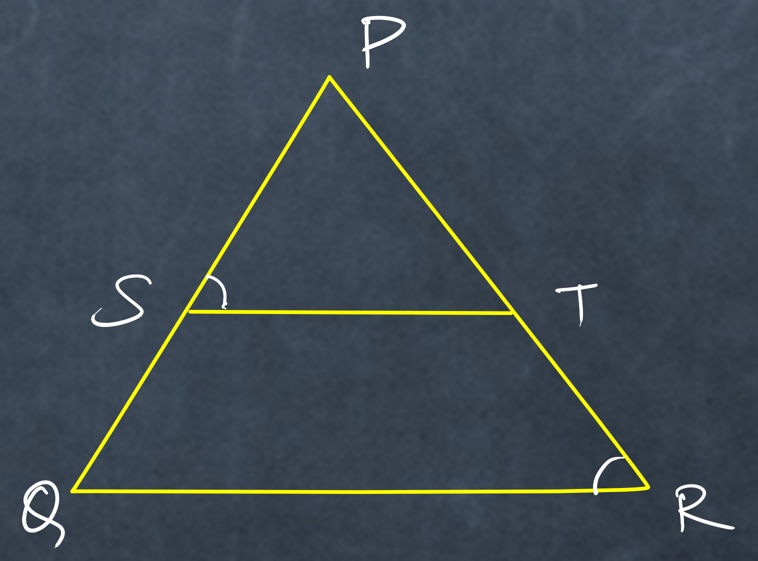


Fig. 6.16



Given: • $\frac{PS}{SQ} = \frac{PT}{TR}$

• $\angle PST = \angle PRQ$

To prove: $\triangle PQR$ is Isosceles

Proof: $ST \parallel QR$ (Converse of BPT)

• $\angle PST = \angle PQR$ ($\because$ Corresponding angles)

but $\angle PST = \angle PRQ$ ($\because$ given)

$\Rightarrow \angle PQR = \angle PRQ \Rightarrow PQ = PR$ ($\because$ sides opp. to equal angles)

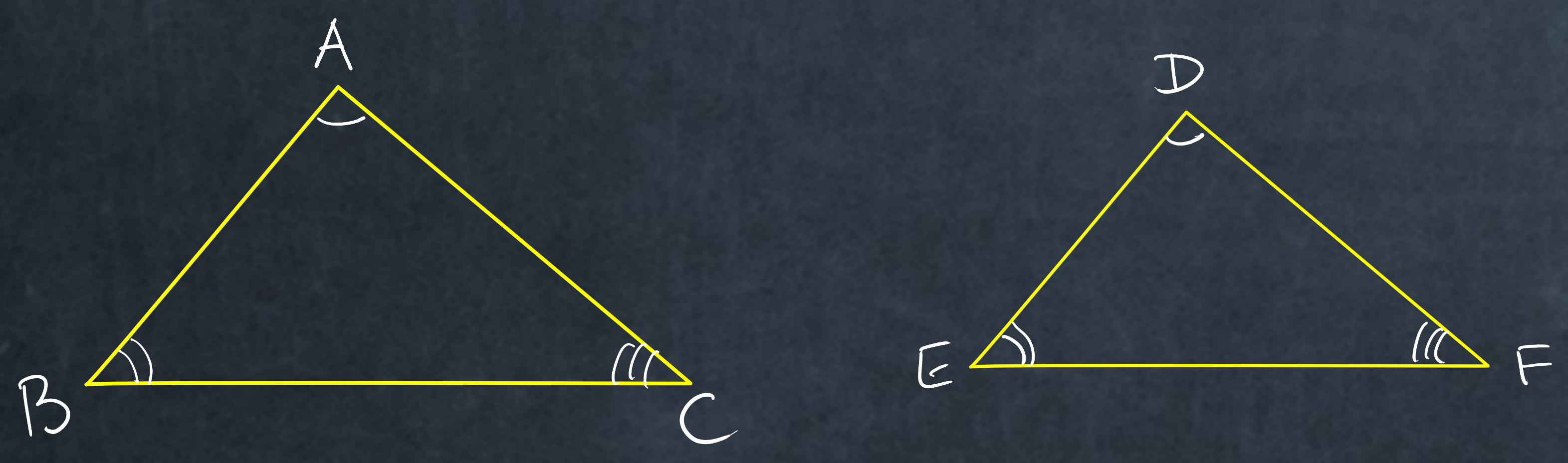
$\therefore$ $\boxed{\triangle PQR \text{ is Isosceles}}$ Hence proved

joel samuel
9182099001

Criteria for Similarity of Triangles

Theorem 6.3 : *If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.*

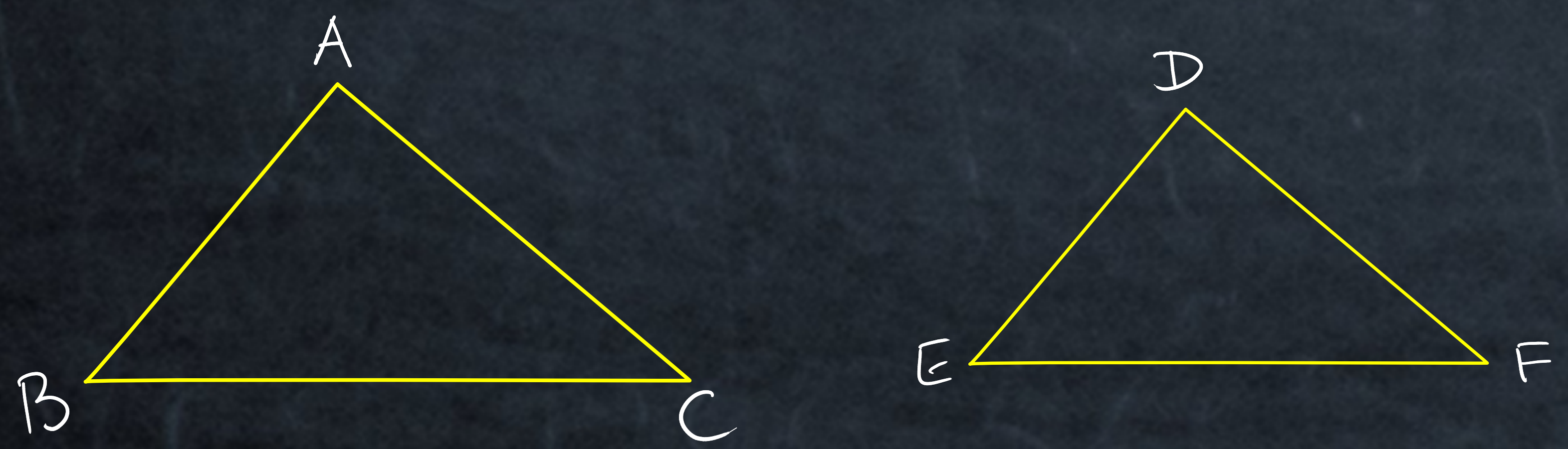
$\Rightarrow$ AAA rule or AA rule



If $\angle A = \angle D$
$\angle B = \angle E$
$\angle C = \angle F$
then $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

Theorem 6.4 : *If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.*

$\Rightarrow$ SSS rule



If $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

then $\angle A = \angle D$
$\angle B = \angle E$
$\angle C = \angle F$

joel samuel
9182099001