

EXERCISE 13.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

MODE: Modal class = 35-45

$$l = 35, f_1 = 23, f_0 = 21, f_2 = 14, h = 10$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h = 35 + \left[\frac{23 - 21}{2(23) - 21 - 14} \right] 10$$

$$\text{Mode} = 36.82$$

CI	f_i	x_i	$f_i x_i$
5-15	6	10	60
15-25	11	20	220
25-35	21	30	630
35-45	23	40	920
45-55	14	50	700
55-65	5	60	300

$$\sum f_i = 80$$

$$\sum f_i x_i = 2830$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.37$$

$$\text{Mean age} = 35.37 \text{ yrs}$$

$$\text{Modal age} = 36.82 \text{ yrs}$$

Interpretation

Avg. age of each patient = 35.37 yrs

Avg. age of max No. of patients = 36.82 yrs.

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components :

Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Modal Class : 60-80

$$l = 60, f_1 = 61, f_0 = 52, f_2 = 38, h = 20$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h = 60 + \left(\frac{61 - 52}{2(61) - 52 - 38} \right) 20$$

$$\text{Mode} = 60 + 5.625$$

$$\text{Mode} = 65.63 \text{ hours}$$

$$\therefore \text{Modal lifetimes} = 65.63 \text{ hours}$$

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in ₹)	Number of families
1000 - 1500	24
1500 - 2000	40
2000 - 2500	33
2500 - 3000	28
3000 - 3500	30
3500 - 4000	22
4000 - 4500	16
4500 - 5000	7

Modal Class : 1500 - 2000

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$l = 1500, f_1 = 40, f_0 = 24, f_2 = 33, h = 500$$

$$\text{Mode} = 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33} \right) 500 = 1847.8$$

Modal monthly expenditure = ₹ 1847.8

CI	f_i	x_i	$d_i = x_i - a$	$u_i = d_i/h$	$f_i u_i$
1000 - 1500	24	1250	-1500	-3	-72
1500 - 2000	40	1750	-1000	-2	-80
2000 - 2500	33	2250	-500	-1	-33
2500 - 3000	28	2750	0	0	0
3000 - 3500	30	3250	500	1	30
3500 - 4000	22	3750	1000	2	44
4000 - 4500	16	4250	1500	3	48
4500 - 5000	7	4750	2000	4	28

$$\bar{x} = a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right)$$

$$= 2750 + 500 \left(\frac{-35}{200} \right)$$

$$\bar{x} = 2662.5$$

Mean monthly
Expenditure = ₹ 2662.5

$$\sum f_i u_i = -35$$

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states / U.T.
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

Modal Class : 30-35

$$l = 30, f_1 = 10, f_0 = 9, f_2 = 3, h = 5$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$= 30 + \left(\frac{10 - 9}{2(10) - 9 - 3} \right) 5$$

$$\text{Mode} = 30.62$$

CI.	f	x_i	$d_i = x_i - a$	$f_i d_i$
15-20	3	17.5	-15	-45
20-25	8	22.5	-10	-80
25-30	9	27.5	-5	-45
30-35	10	32.5	0	0
35-40	3	37.5	5	15
40-45	0	42.5	10	0
45-50	0	47.5	15	0
50-55	2	52.5	20	40

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 32.5 + \frac{(-115)}{35}$$

$$\bar{x} = 29.2$$

$$\text{Mean} = 29.2$$

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches. Find the mode of the data.

Runs scored	Number of batsmen
3000 - 4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7
7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

Modal Class: 4000 - 5000

$$l = 4000, f_1 = 18, f_0 = 4, f_2 = 9, h = 1000$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$$

$$= 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9} \right) 1000$$

$$\text{Mode} = 4608.7 \text{ runs}$$

$\frac{8}{2} \times 10$

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	7	14	13	12	20	11	15	8

Modal Class: 40 - 50

$$l = 40, f_1 = 20, f_0 = 12, f_2 = 11$$

$$h = 10$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h \Rightarrow \text{Mode} = 40 + \left(\frac{20 - 12}{2(20) - 12 - 11} \right) \times 10$$

$$\text{Mode} = 44.7 \text{ cars}$$

Median of Grouped Data

- In case of Grouped data, the median class corresponds to the $\left(\frac{n}{2}\right)^{\text{th}}$ observation.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) h$$

l - Lower limit of median class
 h - class size
 n - No. of observations ($\sum f_i$)
 f - Freq. of median class
 cf - cumulative frequency of preceding class.

- NOTE:**
- To find median class, we find cumulative frequencies of all classes & $\frac{n}{2}$.
 - We now locate the class whose cf is greater than (and nearest to) $\frac{n}{2}$. This is the median class.

EXERCISE 13.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

Median

CI.	f.	cf
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	64
185-205	4	68

$$n = 68 \Rightarrow \frac{n}{2} = 34$$

Median class : 125 - 145

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) h \\ &= 125 + \left(\frac{34 - 22}{20} \right) 20 \end{aligned}$$

$$\text{Median} = 137$$

Mode

Modal Class : 125 - 145

$$l = 125, f_1 = 20, f_0 = 13, f_2 = 14, h = 20$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h \\ &= 125 + \left(\frac{20 - 13}{2(20) - 13 - 14} \right) 20 \end{aligned}$$

$$\text{Mode} = 135.76$$

$$l = 125$$

$$cf = 22$$

$$f = 20$$

$$h = 20$$