Nature of Roots

- Finding roots of a QE. → • By factorization (Split middle term)
  → • The Quadratic formula.

The Quadratic formula : For a QE $ax^2+bx+c=0$, the roots are given by

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$ → Using The Quad. formula.

(i) $x^2 - 3x - 10 = 0$

$a = 1,\ b = -3,\ c = -10$

$$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9+40}}{2} = \frac{3 \pm 7}{2}$$

$$x = \frac{3+7}{2},\ \frac{3-7}{2}$$

$$\boxed{x = 5, -2}$$

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(ii) $2x^2 + x - 6 = 0$

$a = 2$; $b = 1$, $c = -6$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1+48}}{4} = \frac{-1 \pm 7}{4}$$

$$x = \frac{-1+7}{4} , \frac{-1-7}{4}$$

$$\boxed{x = \frac{3}{2} , -2}$$

- To determine Nature of roots, we calculate the Discriminant (D)

$$\Downarrow$$

$$b^2 - 4ac$$

(i) If $b^2 - 4ac > 0 \Rightarrow$ Two real distinct (unequal) roots

(ii) If $b^2 - 4ac = 0 \Rightarrow$ Two real equal roots

(iii) If $b^2 - 4ac < 0 \Rightarrow$ No real roots

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Example 7: Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

$$2x^2 - 4x + 3 = 0$$

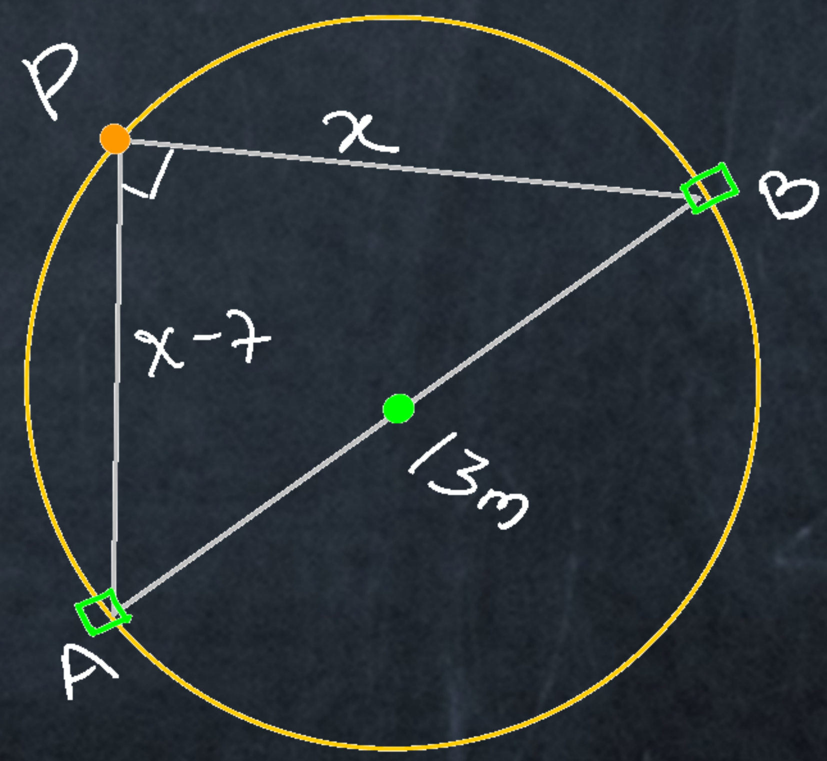
$a = 2$
$b = -4$
$c = 3$

$$b^2 - 4ac = (-4)^2 - 4(2)(3)$$
$$= 16 - 24$$
$$\boxed{b^2 - 4ac = -8 < 0 \Rightarrow \text{No real roots}}$$

R < S
7   14

7

Example 8 : A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?



$\angle APB = 90^\circ$ ($\because$ Diameter subtends 90° on the circle)

By Pythagoras theorem $AP^2 + PB^2 = AB^2$

$$(x-7)^2 + x^2 = 13^2$$
$$x^2 + 49 - 14x + x^2 = 169$$
$$2x^2 - 14x - 120 = 0$$

Divide by 2

$$x^2 - 7x - 60 = 0$$

$$a = 1,\ b = -7,\ c = -60$$

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$$b^2 - 4ac = (-7)^2 - 4(1)(-60)$$

$$b^2 - 4ac = (49 + 240) > 0 \Rightarrow \boxed{\text{It is possible}}$$

$$x = \frac{-(-7) \pm \sqrt{49+240}}{2(1)} = \frac{7 \pm \sqrt{289}}{2} = \frac{7 \pm 17}{2}$$

$$x = \frac{7+17}{2}, \quad \frac{7-17}{2}$$

$x = 12, -5$ — x cannot be -ve since it is dimension

$\therefore x = 12\,m$

$x - 7 = 5\,m$

$\therefore$ $\boxed{\text{Pole must be erected at 12m \& 5m from the two gates}}$

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Example 9 : Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

$$3x^2 - 2x + \frac{1}{3} = 0$$

$$\frac{9x^2 - 6x + 1}{3} = 0 \Rightarrow 9x^2 - 6x + 1 = 0$$

$$a = 9, b = -6, c = 1$$

$$b^2 - 4ac = (-6)^2 - 4(9)(1)$$

$$= 36 - 36$$

$$b^2 - 4ac = 0 \Rightarrow \boxed{\text{Two real equal roots}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)} = \frac{6 \pm 0}{18}$$

$$x = \frac{6}{18} + 0, \frac{6}{18} - 0$$

$$x = \frac{1}{3}, \frac{1}{3}$$

$$\boxed{\text{Roots } \left\{\frac{1}{3}, \frac{1}{3}\right\}}$$

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EXERCISE 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

(i) $2x^2 - 3x + 5 = 0$ $a = 2,\ b = -3,\ c = 5$

Check $b^2 - 4ac$

$b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$

$\therefore$ $\boxed{\text{No real roots}}$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ $a = 3,\ b = -4\sqrt{3},\ c = 4$

Check $b^2 - 4ac$ $= (-4\sqrt{3})^2 - 4(3)(4)$

$(4\sqrt{3})^2$

$= 48 - 48$

$b^2 - 4ac = 0 \Rightarrow$ Two real equal roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{+4\sqrt{3} \pm \sqrt{48-48}}{6} = \frac{2\cancel{4}\sqrt{3}}{3\cancel{6}}$$

$$\boxed{\text{Roots } \left\{ \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3} \right\}}$$

(iii) $2x^2 - 6x + 3 = 0$

$a = 2, \ b = -6, \ c = 3$

$b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = \underline{12} > 0$ Two real distinct Roots

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{-(-6) \pm \sqrt{12}}{2(2)} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$x = \frac{6+2\sqrt{3}}{4}, \ \frac{6-2\sqrt{3}}{4}$$

$$x = \frac{\cancel{2}(3+\sqrt{3})}{\cancel{2}\cdot 2}, \ \frac{\cancel{2}(3-\sqrt{3})}{\cancel{2}\cdot 2} \quad \therefore \boxed{\text{Roots } \left\{ \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2} \right\}}$$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x-2) + 6 = 0$

(i) $2x^2 + kx + 3 = 0$; $a = 2,\ b = k,\ c = 3$

$b^2 - 4ac = 0$ ($\because$ given two equal roots)

$k^2 - 4(2)(3) = 0 \Rightarrow k^2 = 24 \Rightarrow k = \sqrt{24} = \sqrt{4\times6} = 2\sqrt{6}$

$\boxed{k = 2\sqrt{6}}$

(ii) $kx(x-2) + 6 = 0$

$kx^2 - 2kx + 6 = 0$; $a = k,\ b = -2k,\ c = 6$

$b^2 - 4ac = 0$ ($\because$ given two equal roots)

$(-2k)^2 - 4(k)(6) = 0$

$4k^2 - 24k = 0$

$4k(k-6) = 0$

$k = 0$ or $k = 6$

$\boxed{k = 0, 6}$