Example 9 : Express the ratios cos A, tan A and sec A in terms of sin A.

Cos A

From $\sin^2 A + \cos^2 A = 1$

$$\cos^2 A = 1 - \sin^2 A \Rightarrow \boxed{\cos A = \sqrt{1 - \sin^2 A}}$$

tan A

$$\tan A = \frac{\sin A}{\cos A} \Rightarrow \boxed{\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}}$$

Sec A

$$\sec A = \frac{1}{\cos A} \Rightarrow \boxed{\sec A = \frac{1}{\sqrt{1 - \sin^2 A}}}$$

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Example 10 : Prove that sec A (1 – sin A)(sec A + tan A) = 1.

Prove $\sec A(1-\sin A)(\sec A+\tan A)=1$

$$LHS = \left(\sec A - \underline{\sec A \cdot \sin A}\right)(\sec A + \tan A)$$

$$\sec A \cdot \sin A$$

$$\frac{1}{\cos A} \cdot \sin A = \underline{\tan A}$$

$$= (\sec A - \tan A)(\sec A + \tan A)$$

$$= \sec^2 A - \tan^2 A = 1 = RHS$$

Hence proved

Example 11 : Prove that $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

$$LHS = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} = \frac{\frac{\cos A - \cos A \sin A}{\cancel{\sin A}}}{\frac{\cos A + \cos A \sin A}{\cancel{\sin A}}} = \frac{\cos A - \cos A \sin A}{\cos A + \cos A \sin A}$$

$$= \frac{\cancel{\cos A}(1-\sin A)}{\cancel{\cos A}(1+\sin A)} = \frac{1-\sin A}{1+\sin A} = \frac{1 - \frac{1}{\operatorname{cosec} A}}{1 + \frac{1}{\operatorname{cosec} A}} = \frac{\frac{\operatorname{cosec} A - 1}{\cancel{\operatorname{cosec} A}}}{\frac{\operatorname{cosec} A + 1}{\cancel{\operatorname{cosec} A}}}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = RHS$$

Hence proved

Example 12 : Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

$$\frac{x}{a-b} \times \frac{\quad}{a+b}$$

$$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{(1+\sin\theta) - \cos\theta}{(\sin\theta + \cos\theta) - 1}$$

Multiply denominator & Numerator by Conjugate of Denominator

$$= \frac{(1+\sin\theta) - \cos\theta}{(\sin\theta+\cos\theta) - 1} \times \frac{(\sin\theta + \cos\theta) + 1}{(\sin\theta+\cos\theta) + 1} = \frac{[(1+\sin\theta)-\cos\theta][(1+\sin\theta)+\cos\theta]}{(\sin\theta+\cos\theta)^2 - 1^2}$$

$$= \frac{(1+\sin\theta)^2 - \cos^2\theta}{(\sin\theta+\cos\theta)^2 - 1} = \frac{1 + \sin^2\theta + 2\sin\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$$

$$= \frac{(1-\cos^2\theta) + \sin^2\theta + 2\sin\theta}{\cancel{1} + 2\sin\theta\cos\theta - \cancel{1}} = \frac{\sin^2\theta + \sin^2\theta + 2\sin\theta}{2\sin\theta\cos\theta} = \frac{2\sin^2\theta + 2\sin\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2\sin\theta}(\sin\theta + 1)}{\cancel{2\sin\theta}\cos\theta} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$$= \frac{1}{\sec\theta - \tan\theta} = \text{RHS}$$

Hence proved

EXERCISE 8.3

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

$\cosec^2 A - \cot^2 A = 1$

$\sec^2 A - \tan^2 A = 1$

$\sin^2 A + \cos^2 A = 1$

$\cot A = \frac{1}{\tan A}$

Sin A

$$\cosec^2 A - \cot^2 A = 1$$

$$\cosec^2 A = 1 + \cot^2 A$$

$$\cosec A = \sqrt{1+\cot^2 A}$$

$$\boxed{\sin A = \frac{1}{\sqrt{1+\cot^2 A}}}$$

Sec A

$$\sec^2 A - \tan^2 A = 1 \Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\sec A = \sqrt{1+\tan^2 A}$$

$$\boxed{\sec A = \sqrt{1+\frac{1}{\cot^2 A}}}$$

tan A

$$\boxed{\tan A = \frac{1}{\cot A}}$$

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2. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

- $\sin A$ ✓
- $\cos A$ ✓
- $\tan A$ ✓
- $\operatorname{cosec} A$
- $\cot A$

- $$\boxed{\cos A = \frac{1}{\sec A}}$$

- $$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{1}{\sec^2 A}}$$

$$\boxed{\sin A = \sqrt{1-\frac{1}{\sec^2 A}}}$$

- From $\sec^2 A - \tan^2 A = 1$

$$\tan^2 A = \sec^2 A - 1$$

$$\boxed{\tan A = \sqrt{\sec^2 A - 1}}$$

$$\sec^2 A - \tan^2 A = 1$$

$$\tan^2 A =$$

- $$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\sqrt{1-\frac{1}{\sec^2 A}}} = \frac{1}{\sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\boxed{\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}}$$

- $$\cot A = \frac{1}{\tan A} \Rightarrow \boxed{\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}}$$

3. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

$$9\sec^2 A - 9\tan^2 A$$

$$9(\sec^2 A - \tan^2 A) = 9(1) = 9$$

(B) ✓

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(A) 0 (B) 1 (C) 2 (D) –1

$$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{Cosec}\theta)$$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\overbrace{\cos\theta + \sin\theta}^{a} \overbrace{+1}}{\cos\theta}\right)\left(\frac{\overbrace{\sin\theta + \cos\theta}^{a} \overbrace{-1}}{\sin\theta}\right) = \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} = \frac{\cancel{1} + 2\sin\theta\cos\theta - \cancel{1}}{\sin\theta\cos\theta}$$

$$= \frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} = 2$$

(C) ✓

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(iii) (sec A + tan A) (1 – sin A) =

(A) sec A (B) sin A (C) cosec A (D) cos A

$$(\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) = \frac{(1+\sin A)(1-\sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^{\cancel{2}} A}{\cancel{\cos A}} = \cos A$$

D ✓

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A} =$

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

$$\sec^2 A - \tan^2 A = 1$$

$$\operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\cancel{\cos^2 A + \sin^2 A}}{\cos^2 A} \times \frac{\sin^2 A}{\cancel{\sin^2 A + \cos^2 A}}$$

$$= \tan^2 A$$

ALITER

$$\frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} = \frac{1}{\cos^2 A} \cdot \frac{\sin^2 A}{1} = \tan^2 A.$$

D ✓

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